The efficient cultivation of this field of research depends upon combination and assistance of an exceptional kind; but in the first instance money must be available, and the members of the Royal Institution deserve my especial gratitude for their handsome donations to the conduct of this research. Unfortunately its prosecution will demand a further large expenditure. The handsome contribution made by the Goldsmiths Company ought also to be mentioned as very materially helping the progress of the work.

During the whole course of the low temperature work carried out at the Royal Institution, the invaluable aid of Mr. Robert Lennox has been at my disposal; and it is not too much to say that but for his engineering skill, manipulative ability, and loyal perseverance, the present successful issue might have been indefinitely delayed. My thanks are also due to Mr. J. W. Heath for valuable assistance in the conduct of these experiments.

## "Effects of Prolonged Heating on the Magnetic Properties of Iron." By S. R. Roget, B.A. Communicated by Professor Ewing, F.R.S. Received April 4,—Read May 12, 1898.

It has been known for some years that when transformers are kept in use, their open-circuit loss is liable to increase considerably with the lapse of time. This implies a deterioration of the iron core in regard to magnetic hysteresis. The subject began to receive attention in 1894–5, when some curves showing this increase in hysteresis were published by Mr. G. W. Partridge.* The effect was first thought to be due to a species of magnetic fatigue, resulting from repeated reversals of magnetism in the iron, but it was proved by Professor Ewing early in 1895 not to be due to this cause;† and was further shown by the experiments of Mr. Bláthy and Mr. Mordey to be a direct effect of heat and to occur in transformers as a consequence of the iron being maintained for long periods at a comparatively high temperature. Continued baking of iron was found to produce a similar augmentation of hysteresis. The published results of Mr. Mordey,‡ and, later, those of Mr. Parshall,§ deal with prolonged heating at temperatures which do not exceed 140° C. At the suggestion of Professor Ewing, the author has been carrying out, in the laboratory of the Engineering Department of Cambridge University, some investigations which deal with a more extended range of temperature. The experiments are still in progress, but

* 'The Electrician,' vol. 34, p. 160, December, 1894.

† *Ibid.*, p. 297, January, 1895.

‡ 'Proc. Roy. Soc.,' vol. 57, p. 224, June, 1895.

§ 'Min. Proc. Inst. C.E.,' vol. 126, p. 244.

already results have been obtained which appear to be of sufficient novelty to warrant publication.

The hysteresis of the iron was directly measured by means of Professor Ewing's Hysteresis Tester, in which the work spent on a specimen rotating in a magnetic field is observed and is compared with the work spent in rotating standard specimens.* In these experiments the same pair of standards was used throughout for the calibration of the hysteresis tester. The test specimens were all cut from the same sheet of metal and were of soft Swedish transformer plate, having very low initial hysteresis. They were first tested in the annealed state and were then heated in small ovens which were kept hot by means of incandescent lamps. The temperatures of the ovens were observed in most cases by mercury thermometers, but those above 200° C. were measured by a Callendar-Griffiths platinum pyrometer. The specimens were taken out of the ovens from time to time to be tested, and all the tests of hysteresis were made at atmospheric temperature. It was not found possible to keep the temperature of each oven very constant, but when the ovens were once hot, the variation of temperature was rarely more than 10 degrees C. in either direction. To these variations may be ascribed certain irregularities which will be apparent in the observations, but the general character of the changes due to prolonged heating is sufficiently clear. Each specimen consisted of a bundle of seven strips 3 inches long, and about $\frac{5}{8}$ inch wide, and each strip was annealed separately by heating it to redness in a Bunsen flame, and allowing it to cool in the air. As the effects of prolonged heating described below were in all cases found to be completely removed by reannealing, the same samples could be used over and over again, and this was in fact done in most cases. In all the experiments the measurement of hysteresis relates to cyclic processes in which the induction B changes from + 4000 to − 4000 C.G.S. units.

The effects produced by baking differ widely at different temperatures. Below 40° C. the author has found no evidence of any change. Between 40° C. and about 135° C. the hysteresis simply increases with time, at least during the longest time of heating tried in these experiments. The increase of hysteresis is relatively rapid at first, and becomes slower as times goes on. Curves 1—4, fig. 1 show results of this nature by exhibiting the percentage increase in hysteresis after various times of baking. The absolute values of the hysteresis at the different stages are stated in Table I in ergs per cycle per cubic centimetre (for $B = 4000$) together with the rise expressed as a percentage of the initial hysteresis to the nearest

* 'Journal Inst. Elect. Eng.,' vol. 24, p. 403; also 'Min. Proc. Inst. C.E.,' vol. 126, p. 206.

1 per cent. The curves have been sketched by joining the observed points instead of drawing smooth curves through them, as this avoids confusion of points belonging to different curves.

It was found however that at higher temperatures, from about 135° C. upwards, a maximum value of the hysteresis was attained in a comparatively short time, after which continued heating caused a marked *decrease* of hysteresis instead of a further increase. The initial rise at the higher temperatures is very rapid; for example, the hysteresis doubles in a few hours at a temperature of 160° C., and reaches nearly three times its initial value in a few days. Curve 5 of fig. 1 exhibits this case, the data for which are given in Table I. After seven days of heating, the hysteresis of this sample began to decrease, and in fifteen days it had fallen to $2\frac{1}{2}$ times its original value. A still more notable decrease occurs at higher

FIG. 1.

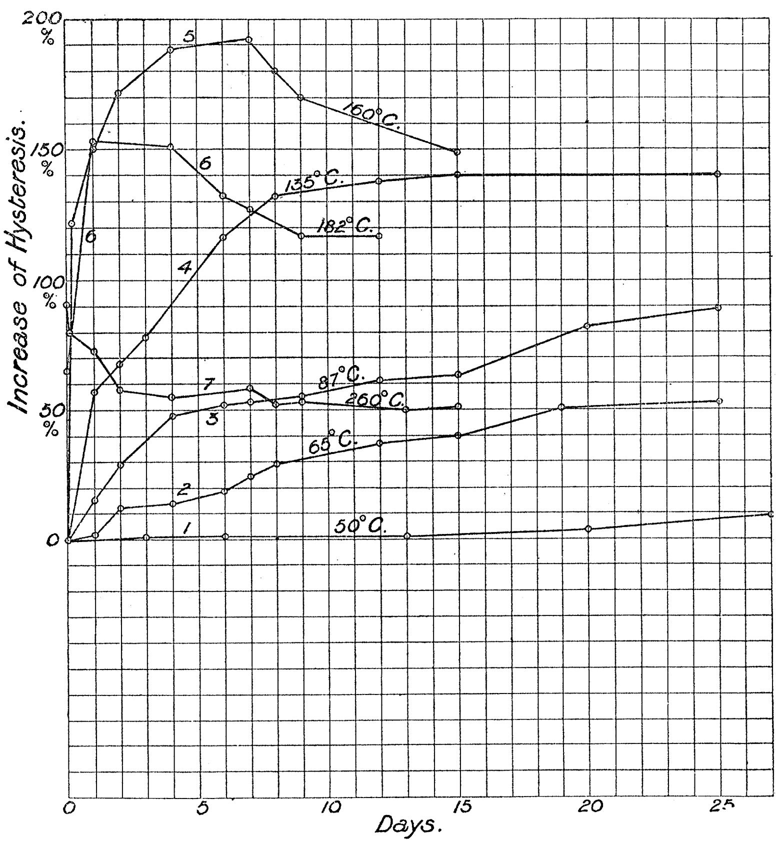
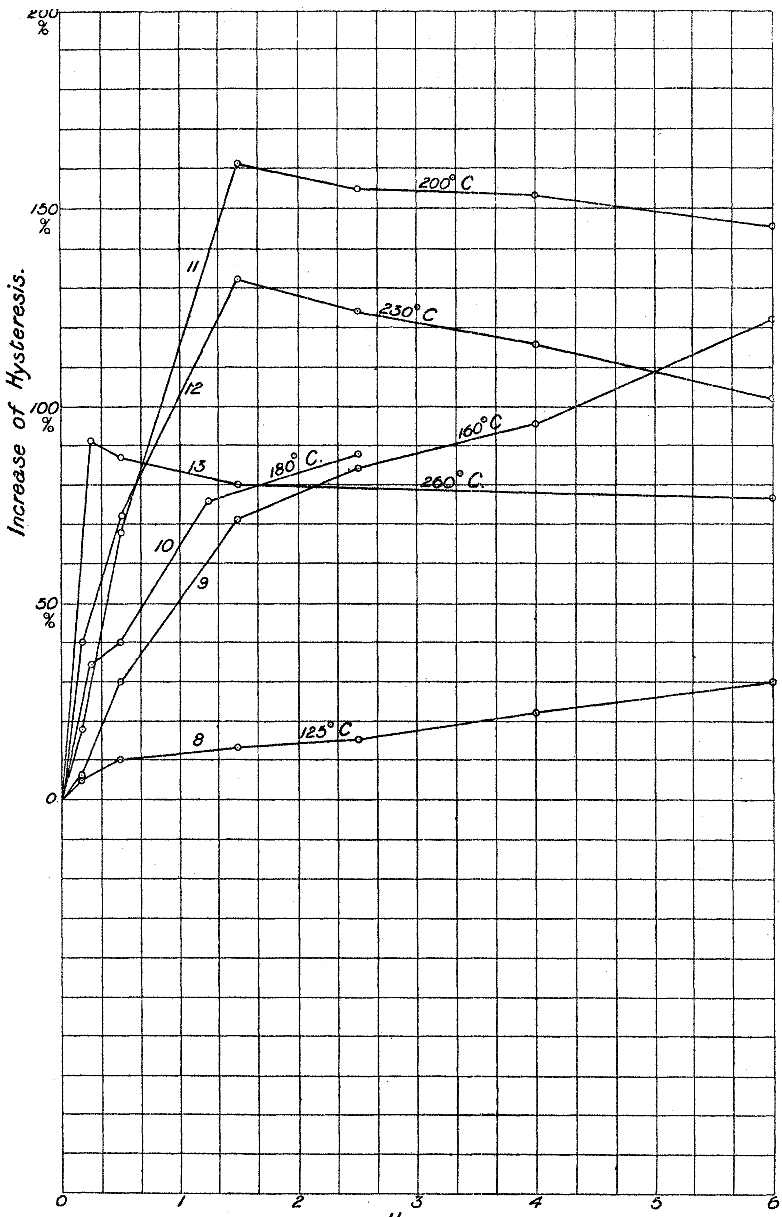


FIG. 2.



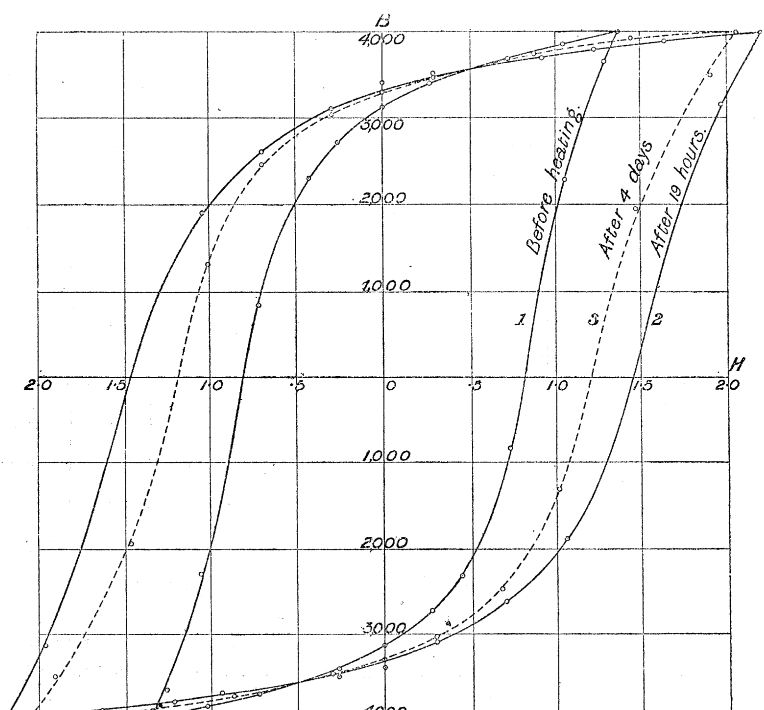

temperatures. This feature in the effect of prolonged heating seems to have escaped the notice of previous workers in the subject.

It appears that there is a temperature in the neighbourhood of 180° C., for which the maximum increase of hysteresis is greatest. With higher temperatures the hysteresis, although rising more rapidly at first, does not reach so high a maximum value and begins to fall sooner and faster, tending apparently to a lower steady state the higher the temperature. An example of this is shown in curve 7, fig. 1 (temperature 260° C.), where a fairly low and nearly steady state is reached in the last days of the heating. In this instance it took the iron only about a quarter of an hour to reach its maximum of hysteresis, which was only 91 per cent. higher than the initial value.

Fig. 2 shows the earlier stages of the action for temperatures of 125° C. and over. It will be noticed that the peak at which the hysteresis reaches its maximum in each case comes sooner the higher the temperature, and that its height becomes reduced when the temperature is high. The absolute values of the hysteresis in the experiments to which these curves relate are given in Table II.

It is probable that the attainment of a maximum value followed by a decrease is not confined to temperatures above 135° C., and it

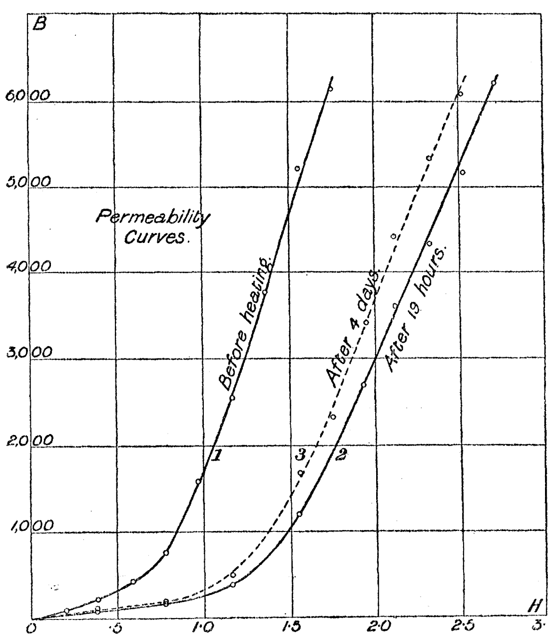
Fig. 3.



is intended to carry out experiments to find if this is so, using more prolonged heating.

In order to exhibit the character of the change in magnetic property, supplementary experiments were made by the ballistic method, using a ring of soft iron formed by coiling up a long strip of sheet metal. This was first annealed, and tested in the annealed state. It was then baked by heating at 200° C., and cyclic curves were determined in the usual way after the ring had become cold. The results are stated in Table III, and are shown in the curves of fig. 3. Curve 1 shows the initial state (after annealing), where the value of the hysteresis is 830 ergs per cycle per cubic centimetre (B = 4000). Curve 2 shows the state after nineteen hours of baking at 200° C., when the hysteresis had greatly increased and had reached a value of 1580 ergs. Curve 3 was taken after further heating at the same temperature for four days, by which time the decrease of hysteresis is very apparent, its value having diminished to 1420 ergs. Permeability curves, taken by the method of reversals after heating during the same periods, are given in fig. 4 and

FIG. 4.




show the falling off and subsequent partial recovery in the permeability.

Table I.—Change of Hysteresis by prolonged Heating.

| Curve No. . | 1. | | 2. | | 3. | | 4. | | 5. | | 6. | | 7. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temperature | 50° C. | | 65° C. | | 87° C. | | 135° C. | | 160° C. | | 182° C. | | 260° C. | |
| Time in days. | Hysteresis. | | Hysteresis. | | Hysteresis. | | Hysteresis. | | Hysteresis. | | Hysteresis. | | Hysteresis. | |
| | Abs. C.G.S. | Increase per cent. | Abs. C.G.S. | Increase per cent. | Abs. C.G.S. | Increase per cent. | Abs. C.G.S. | Increase per cent. | Abs. C.G.S. | Increase per cent. | Abs. C.G.S. | Increase per cent. | Abs. C.G.S. | Increase per cent. |
| 0 | 635 | 0 | 620 | 0 | 600 | 0 | 610 | 0 | 590 | 0 | 665 | 0 | 595 | 0 |
| 1 | .. | .. | 635 | 2 | 695 | 16 | 960 | 57 | 1480 | 151 | 1680 | 153 | 1030 | 73 |
| 2 | .. | .. | 695 | 12 | 770 | 29 | 1020 | 67 | 1600 | 172 | .. | .. | 940 | 58 |
| 3 | 643 | 1·3 | .. | .. | .. | .. | 1090 | 78 | .. | .. | .. | .. | .. | .. |
| 4 | .. | .. | 710 | 14 | 885 | 48 | .. | .. | 1700 | 188 | 1670 | 151 | 920 | 55 |
| 6 | 645 | 1·6 | 740 | 19 | 910 | 52 | 1325 | 117 | .. | .. | 1540 | 132 | 940 | 58 |
| 7 | .. | .. | 775 | 25 | 915 | 53 | .. | .. | 1720 | 192 | 1515 | 128 | 905 | 52 |
| 8 | .. | .. | 805 | 29 | .. | .. | 1415 | 132 | 1650 | 180 | .. | .. | .. | .. |
| 9 | .. | .. | .. | .. | 930 | 55 | .. | .. | 1590 | 170 | 1445 | 117 | 910 | 53 |
| 12 | .. | .. | 855 | 37 | 960 | 61 | 1450 | 138 | .. | .. | 1440 | 116 | .. | .. |
| 13 | 645 | 1·6 | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 890 | 50 |
| 15 | .. | .. | 875 | 40 | 975 | 63 | 1465 | 140 | 1470 | 149 | .. | .. | 900 | 51 |
| 20 | 660 | 4 | 940 | 51 | 1090 | 82 | .. | .. | | | | | | |
| 25 | .. | .. | 945 | 53 | 1135 | 89 | 1465 | 140 | | | | | | |
| 27 | 690 | 9 | | | | | | | | | | | | |

Table II.—Change of Hysteresis by prolonged Heating; early stages.

| Curve No. . . . . . | 8. | | 9. | | 10. | | 11. | | 12. | | 13. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temperature . . | 125° C. | | 160° C. | | 180° C. | | 200° C. | | 230° C. | | 260° C. | |
| Time of heating. | Hysteresis. | | Hysteresis. | | Hysteresis. | | Hysteresis. | | Hysteresis. | | Hysteresis. | |
| | Abs. | Increase per cent. | Abs. | Increase per cent. | Abs. | Increase per cent. | Abs. | Increase per cent. | Abs. | Increase per cent. | Abs. | Increase per cent. |
| Hrs. mins. 0 0 | 600 | 0 | 595 | 0 | 590 | 0 | 570 | 0 | 570 | 0 | 595 | 0 |
| 0 10 | 630 | 5 | 630 | 6 | .. | .. | 785 | 18 | 800 | 40 | .. | .. |
| 0 15 | .. | .. | .. | .. | 790 | 33 | .. | .. | .. | .. | 1145 | 91 |
| 0 30 | 660 | 10 | 770 | 30 | 825 | 40 | 960 | 68 | 980 | 72 | 1110 | 87 |
| 1 15 | .. | .. | .. | .. | 1040 | 76 | .. | .. | .. | .. | .. | .. |
| 1 30 | 680 | 13 | 1015 | 71 | .. | .. | 1480 | 160 | 1320 | 132 | 1070 | 80 |
| 2 30 | 690 | 15 | 1090 | 84 | 1100 | 87 | 1455 | 155 | 1275 | 124 | .. | .. |
| 4 0 | 730 | 22 | 1160 | 96 | .. | .. | 1440 | 153 | 1230 | 116 | .. | .. |
| 6 0 | 800 | 30 | 1300 | 122 | .. | .. | 1415 | 146 | 1080 | 102 | .. | .. |

Table III.—Cyclic Curves for Ring.

| 1. Before heating. | | 2. After 20 hours at 200° C. | | 3. After 4 days at 200° C. | |
|---|---|---|---|---|---|
| H. | B. | H. | B. | H. | B. |
| 1·36 | 4000 | 2·20 | 4000 | 2·06 | 4000 |
| 1·26 | 3650 | 1·97 | 3170 | 1·91 | 3500 |
| 1·06 | 2300 | 1·05 | –1900 | 1·47 | 1950 |
| 0·72 | –850 | 0·70 | –2620 | 1·02 | –1310 |
| 0·44 | –2320 | 0·28 | –3100 | 0·68 | –2480 |
| 0·26 | –2740 | 0·00 | –3400 | 0·28 | –3050 |
| 0·00 | –3130 | –0·28 | –3460 | 0·00 | –3310 |
| –0·26 | –3400 | –0·87 | –3700 | –0·28 | –3520 |
| –0·72 | –3700 | –1·23 | –3790 | –0·87 | –3740 |
| –1·05 | –3850 | –1·51 | –3880 | –1·45 | –3930 |

Table IV.—Permeability Curves for Ring.

| 1. Before heating. | | 2. After 20 hours at 200° C. | | 3. After 4 days at 200° C. | |
|---|---|---|---|---|---|
| H. | B. | H. | B. | H. | B. |
| 0·19 | 90 | 0·39 | 70 | 0·39 | 100 |
| 0·39 | 220 | 0·78 | 160 | 0·78 | 190 |
| 0·58 | 420 | 1·16 | 390 | 1·16 | 520 |
| 0·78 | 760 | 1·55 | 1200 | 1·55 | 1660 |
| 0·97 | 1570 | 1·94 | 2730 | 1·75 | 2320 |
| 1·16 | 2550 | 2·12 | 3600 | 1·94 | 3400 |
| 1·36 | 3760 | 2·33 | 4320 | 2·12 | 4420 |
| 1·55 | 5220 | 2·52 | 5480 | 2·33 | 5340 |
| 1·75 | 6150 | 2·72 | 6220 | 2·52 | 6100 |

In order to ascertain whether the effects could be due to the iron taking up anything from the atmosphere, a couple of specimens were, at the suggestion of Professor Ewing, sealed up in exhausted bulbs and heated in the same oven with others exposed to air. No evidence, however, was found of any difference between the iron heated *in vacuo*, and that heated in air.

There is nothing to show that when baked iron is left for a long time it recovers from the condition produced by baking. But in all cases the iron is restored to its original state of low hysteresis by reannealing.

In conclusion the author wishes to express his thanks to Professor Ewing for many suggestions as well as for the facilities which have enabled the experiments to be carried out.

[Subsequent to the writing of the above paper, permeability curves have been taken, for a ring of the same material, by the ballistic method, up to a magnetic force of 25 C.G.S. units. A comparison of those taken before and after baking shows that the saturation value of the induction is unchanged by prolonged heating, for although the earlier part of the curve, as in the case given above, was much altered, the parts of the curves above a force of 15 C.G.S. units are practically indistinguishable.—*May* 15, 1898.]

## "On the Connexion of Algebraic Functions with Automorphic Functions." By E. T. WHITTAKER, B.A., Fellow of Trinity College, Cambridge. Communicated by Professor A. R. FORSYTH, Sc.D., F.R.S. Received April 23,—Read May 12, 1898.

(Abstract.)

If $u$ and $z$ are variables connected by an algebraic equation, they are, in general, multiform functions of each other; the multiformity can be represented by a Riemann surface, to each point of which corresponds a pair of values of $u$ and $z$.

Poincaré and Klein have proved that a variable $t$ exists, of which $u$ and $z$ are uniform automorphic functions; the existence-theorem, however, does not connect $t$ analytically with $u$ and $z$. When the genus (*genre*, *Geschlecht*) of the algebraic relation is zero or unity, $t$ can be found by known methods; the automorphic functions required are rational functions, and doubly periodic functions, in the two cases respectively. But no class of automorphic functions with simply connected fundamental polygons has been known hitherto, which is applicable to the uniformisation of algebraic functions whose genus is greater than unity.

The present memoir discusses a new class of groups of projective substitutions, such that the functions rational on a Riemann surface of any genus can be expressed as uniform automorphic functions of a group of this class. These groups are sub-groups of groups generated from substitutions of period two. Groups are first considered which can be generated by a number of real substitutions of period two, whose double points are not on the real axis, and whose product in a definite order is the identical substitution. These groups are found to be discontinuous, and of genus zero. A method is given for