In conclusion the author wishes to express his thanks to Professor Ewing for many suggestions as well as for the facilities which have enabled the experiments to be carried out.

[Subsequent to the writing of the above paper, permeability curves have been taken, for a ring of the same material, by the ballistic method, up to a magnetic force of 25 C.G.S. units. A comparison of those taken before and after baking shows that the saturation value of the induction is unchanged by prolonged heating, for although the earlier part of the curve, as in the case given above, was much altered, the parts of the curves above a force of 15 C.G.S. units are practically indistinguishable.—*May* 15, 1898.]

"On the Connexion of Algebraic Functions with Automorphic Functions."

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(Abstract.)

If u and z are variables connected by an algebraic equation, they are, in general, multiform functions of each other; the multiformity can be represented by a Riemann surface, to each point of which corresponds a pair of values of u and z.

Poincaré and Klein have proved that a variable t exists, of which u and z are uniform automorphic functions; the existence-theorem, however, does not connect t analytically with u and z. When the genus (*genre*, *Geschlecht*) of the algebraic relation is zero or unity, t can be found by known methods; the automorphic functions required are rational functions, and doubly periodic functions, in the two cases respectively. But no class of automorphic functions with simply connected fundamental polygons has been known hitherto, which is applicable to the uniformisation of algebraic functions whose genus is greater than unity.

The present memoir discusses a new class of groups of projective substitutions, such that the functions rational on a Riemann surface of any genus can be expressed as uniform automorphic functions of a group of this class. These groups are sub-groups of groups generated from substitutions of period two. Groups are first considered which can be generated by a number of real substitutions of period two, whose double points are not on the real axis, and whose product in a definite order is the identical substitution. These groups are found to be discontinuous, and of genus zero. A method is given for

dividing the plane into curvilinear polygons corresponding to such a group; these polygons are simply-connected, and cover completely the half of the plane which is above the real axis. Sub-groups of these groups are found, whose genus is greater than unity, and which are appropriate for the uniformisation of any algebraic curves.

The sides of the polygons, into which the half-plane is divided, are formed of arcs of circles orthogonal to the real axis. These may, in the sense of Lobatchewski's geometry, be regarded as straight lines. One case, where the construction fails, is shown to correspond to the limiting case in which Lobatchewski's geometry becomes Euclidian geometry; the figure then becomes the division of a plane into parallelograms, used in the theory of doubly periodic functions, and is appropriate for the uniformisation of algebraic curves of genus unity. Thus doubly-periodic functions are a limiting case of the class of functions considered.

The automorphic functions of the groups described solve the problem of conformally representing a plane, regarded as bounded by a number of finite lines radiating from a point, on a curvilinear polygon, whose sides are derived from each other in pairs by projective substitutions of period two. This leads to the conformal representation of any Riemann surface, at each of whose branch-points only two sheets are connected, on a curvilinear polygon whose sides are derived from each other in pairs by projective substitutions; and, as it is known that any algebraic curve can, by birational transformation, be represented on a Riemann surface whose branch-points are all simple, it is seen that the uniformisation of algebraic functions of any genus can be effected by groups of the kind described.

The analytical connexion between the variables of the algebraic form and the uniformising variables is given by a differential equation of the third order. A certain number of the constants in this equation have to be determined by the condition that the group of substitutions associated with the equation leaves unchanged a certain circle. When any arbitrary values are given to these constants the solution of the differential equation is termed a quasi-uniformising variable. The properties of quasi-uniformising variables, and their relation to the uniformising variable, are discussed in the last section of the paper.