

From this it results that the true azimuth			
of the Sun at the time of observation = N.	50°	30'	54" E.
And since azimuth of Friar's Heel ..... =	50	39	5
2' of sunrise should be N. of Friar's Heel	0	8	11
Observed difference of azimuth ..... =	0	8	40
Observed—calculated ..... =	0	0	29

The observation thus agrees with calculation, if we suppose about 2' of the Sun's limb to have been above the horizon when it was made, and therefore substantially confirms the azimuth above given of the Friar's Heel and generally the data adopted.

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“The pear-shaped Figure of Equilibrium of a Rotating Mass of Liquid.” By G. H. DARWIN, F.R.S., Plumian Professor and Fellow of Trinity College, Cambridge. Received October 21, 1901.

(Abstract.)

This is the sequel to a paper on “Ellipsoidal Harmonic Analysis,” presented to the Royal Society in June, 1901.

Rigorous expressions for the harmonics of the third degree may be found by the methods of that paper, and the processes are carried out here. The functions of the second kind are also found, and are expressed in elliptic integrals.

So much of the results of M. Poincaré's celebrated memoir\* on rotating liquid as relates to the immediate object in view is re-investigated, with a notation adapted for the use of the harmonics already determined. The general expressions for the coefficients of stability having been found, those for the seven coefficients corresponding to the harmonics of the third degree, as applicable to the Jacobian ellipsoids, are reduced to elliptic integrals.

The principal properties of these coefficients, as established by M. Poincaré, are enumerated. He has shown that the ellipsoid can bifurcate only into figures defined by zonal harmonics with reference to the longest axis of the Jacobian ellipsoid; that it must do so for all degrees; and that the first bifurcation occurs with the third zonal harmonic.

A numerical result given in the paper seems to indicate that as the ellipsoid lengthens, it becomes more stable as regards deformations of the third degree and of higher orders, and less stable as regards the lower orders of the same degree.

\* ‘Acta Math.,’ vol. 7, 1885.

The numerical solution of the equation furnished by the vanishing of the coefficient corresponding to the third zonal harmonic shows that the critical Jacobian ellipsoid is such that its axes are proportional to 0.65066, 0.81498, 1.88583; and that the angular velocity  $\omega$  and density  $\rho$  of the liquid are connected by the equation  $\frac{\omega^2}{2\pi\rho} = 0.14200$ .

This ellipsoid is the longest stable figure of Jacobi's series. A figure of the deformation of this critical ellipsoid by the third zonal harmonic is delineated in a plate. The so-called pear-shaped figure is seen to be longer than was indicated by M. Poincaré in his conjectural sketch.

Although this figure is almost certainly stable, absolute proof is still wanting. This proof can only be obtained by proceeding to a higher degree of approximation. An attempt is made to obtain this higher approximation, and the cause of failure and the difficulties of the problem are discussed.

“Sur la Stabilité de l'Équilibre des Figures Pyriformes affectées par une Masse Fluide en Rotation.” By H. POINCARÉ, Foreign Member R.S. Received October 29, 1901.

(Abstract.)

J'ai publié autrefois dans le Tome 7 des 'Acta Mathematica' un mémoire où j'étudie diverses figures d'équilibre nouvelles d'une masse fluide homogène en rotation. Presque toutes ces figures sont instables; une d'elles cependant, qui est pyriforme, est très probablement stable. Mais la preuve directe de cette stabilité ne pourrait être obtenue que par de longs calculs. Le but du présent travail est de faciliter ces calculs, en donnant à la condition de stabilité une forme analytique aussi simple que possible. La question cependant reste indécise, parce que les formules analytiques n'ont pas été réduites en chiffres.

Il fallait d'abord obtenir une expression de l'énergie de gravitation d'une pareille figure en poussant l'approximation plus loin qu'on ne l'avait fait jusqu'ici. L'emploi des fonctions de Lamé peut conduire au résultat, mais on se trouve en présence d'une petite difficulté. Le potentiel d'un ellipsoïde, ou d'une couche ellipsoïdale, affecte des formes analytiques différentes selon que le point envisagé est à l'intérieur ou à l'extérieur de l'ellipsoïde. Il en résulte que dans chacune des intégrales il faudrait donner à la fonction sous le signe  $\int$ , tantôt une forme pour les parties de la surface pyriforme qui sont au dessous de la surface de l'ellipsoïde, tantôt une autre forme pour les parties qui sont au dessus. Mais j'ai reconnu que cette difficulté est purement artificielle et qu'on obtiendra encore un résultat final correct en