"On the Theory of Consistence of Logical Class-frequencies and its Geometrical Representation." By G. UDNY YULE, formerly Assistant Professor of Applied Mathematics in University College, London. Communicated by Professor K. PEARSON, F.R.S. Received February 9,—Read February 28, 1901.

(Abstract.)

The memoir deals with the theory of the conditions to which a series of logical class-frequencies is subject if the series is to be self-consistent; *i.e.*, if the class-frequencies are to be such as might be observed within one and the same logical universe.

The theory has been dealt with to a limited extent by De Morgan, in his 'Formal Logic' ("On the Numerically Definite Syllogism") and by Boole, in the 'Laws of Thought' (in the chapter entitled "Of Statistical Conditions").

In the present memoir the first section deals with the theory of consistence, by a simple method, up to class-frequencies in five attributes, and a general formula is then obtained, giving the conditions for any case. In the second part of the paper some illustrations are given of the geometrical representations of the conditions obtained in Part I.

In the case of three second-order frequencies (AB), (AC), and (BC), the complete conditions of consistence may be represented by a tetrahedron with its edges truncated. The first-order frequencies are treated as constant, (AB), (AC), (BC) as co-ordinates, and the limits to (BC), for example, are given by the points in which the line drawn through the point (AB) (AC) parallel to the (BC)-axis cuts the surface. The general form of the surface depends on the value of the first-order frequencies. If

$$(A)/(U) = (B)/(U) = (C)/(U) = \tfrac{1}{2}$$

(U) being the total frequency, the edges are not truncated and the "congruence-surface" becomes a simple equilateral tetrahedron. The limits given to (BC) in terms of (AB) and (AC) in this case are shown to correspond to the limits to the correlation coefficient r_{23} in terms of r_{12} and r_{13} in the case of normal correlation. The congruence-surface shows very clearly the nature of the approximation towards the syllogism, as conditions of the "universal" type (all A's are B, or no A's are B) are approached. One or two illustrations are also given of congruence-surfaces for third-order frequencies, the first- and second-order frequencies being both treated as constants.

In the third part of the paper some numerical examples, and sketches of congruence-surfaces for actual cases, are given, in further illustration of the theory.