30·9, 61·8: these values are expressed in the curve of fig. 5, which is therefore a curve expressing roughly the luminosity of different parts of the same spectrum according to the evidence afforded by flicker. For the sake of comparison the luminosity of the different colours as measured by Vierordt is indicated by a dotted curve, and it will be seen that the two curves practically coincide, except for the extreme parts of the spectrum; it should, however, be stated that Vierordt's curve has been drawn by making his value for the luminosity of the yellow coincide with that given for this colour by the continuous curve in the figure, and reducing his other values in the same proportion: the reduced values are then placed within the limits of the colours to which they are assigned by Vierordt. Moreover, Vierordt's curve is for a solar prismatic spectrum, whilst the other is for incandescent lime, used with a grating.* Lastly it should be remembered that the disc was very feebly illuminated in the violet and extreme crimson, and also, for the small values of the white coloured sector, in the blue, blue-green, and vermilion, so that the alteration of the relation between n and log I (indicated in fig. 2 by the steeper line for feeble illuminations) should probably be considered: if this is so, the two curves will be brought into still closer union. The writer postpones the consideration of this matter, until he has had the opportunity of trying further experiments with such feeble illuminations.

"The Refractive Indices of Fluorite, Quartz, and Calcite." By J. WILLIAM GIFFORD. Communicated by SILVANUS P. THOMPSON, F.R.S. Received February 5,—Read February 13,—Received in revised form May 7, 1902.

1. *Method of Observation.*—Measurements of fluorite, quartz, and calcite have been made by Rudberg, Mascart, Cornu, Sarasin, Glazebrook, Van der Willigen, Vogel, Pulfrich, Rubens, Baille, and many others. Those now offered were originally undertaken with the view of further extending the range and accuracy of lenses constructed of these substances. A new method of obtaining the refractive indices has been adopted. Each of the angles of the prisms used was as nearly as possible 60°. When this is the case it is sufficient to measure the deviation of light of a definite wave-length at each angle in turn; the mean of these deviations may be taken as the deviation corresponding

* C. Vierordt, 'Pogg. Ann.,' vol. 137, p. 200.

to an angle of 60°, and if D denote this mean, the refractive index $\bar{\mu}$ is given very approximately by the formula*

$$\bar{\mu} = \sin \tfrac{1}{2} (\mathrm{D} + 60°)/\sin 30°.$$

The method has several advantages. In the first place it is not necessary to measure the angles of the prism with accuracy; then, again, if the prism be suitably placed, light reflected from the outside of the base enters the telescope and is only parallel to the rays of the wave-length under measurement if those rays have passed through the prism parallel to its base. Thus we have two images of the slit, one by refraction the other by reflection, in the field of view in the same direction when the condition for minimum deviation is satisfied. When they overlap in the field of view of the eye-piece, we may, therefore, rest assured that we have minimum deviation for the wave-length under observation. By sliding the prism in a direction at right angles to its base, it is easy to regulate the amount of light thus reflected from the outer side of the base.

2. *Instruments.*—A special spectrometer by Hilger with objectives of quartz, the collimator provided with bars carrying the spark apparatus

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* The following investigation by Dr. Glazebrook will show the amount of error introduced by the method:—

Let D be the deviation and A the angle of the prism; and let D + δ be the deviation corresponding to the angle A + α. Then we have

$$\frac{\sin \frac{\mathrm{D}+\mathrm{A}}{2}}{\sin \frac{\mathrm{A}}{2}} = \mu = \frac{\sin \frac{\mathrm{D}+\delta+\mathrm{A}+\alpha}{2}}{\sin \frac{\mathrm{A}+\alpha}{2}},$$

whence we find, if α be small,

$$\delta = \alpha \cdot \frac{\sin \frac{\mathrm{D}}{2}}{\sin \frac{\mathrm{A}}{2} \cos \frac{\mathrm{D}+\mathrm{A}}{2}} + \frac{\alpha^2}{4} \cdot \frac{\sin \frac{\mathrm{D}+2\mathrm{A}}{2} \sin \frac{\mathrm{D}}{2}}{\sin^2 \frac{\mathrm{A}}{2} \cos^2 \frac{\mathrm{D}+\mathrm{A}}{2}} \tan \frac{\mathrm{D}+\mathrm{A}}{2}.$$

Now, if α_1, α_2, α_3 be the differences from 60° of the three angles of one of the prisms, each measured in circular measure, D, the deviation which would be observed if the angle were 60°, and D + δ_1, D + δ_2, D + δ_3, the actually observed deviations, then A is 60° and $\alpha_1 + \alpha_2 + \alpha_3$ is zero. Hence, if $\bar{\mu}$ be the refractive index as found by the method adopted in the paper, and μ the true refractive index, we have

$$\mu = \frac{\sin \frac{1}{2}(\mathrm{D}+\mathrm{A})}{\sin \frac{1}{2}\mathrm{A}}, \quad \bar{\mu} = \frac{\sin \frac{1}{2}\{\mathrm{D}+\mathrm{A}+\frac{1}{3}(\delta_1+\delta_2+\delta_3)\}}{\sin \frac{1}{2}\mathrm{A}};$$

and a condensing lens also of quartz, the whole of more than ordinarily solid construction, was used for the work. On this instrument each degree of the circle is subdivided into 12 parts (of 5′ each), and the wheel of the micrometer, one revolution of which corresponds with one subdivision of the circle, is divided into 5 parts (of 1′ each), and subdivided into 300 parts (of 1″ each), thus rendering the measurement of seconds of arc, and with care fractions of seconds, possible. Readings were always taken to 0·25″, the quartz fibre of the micrometer being brought into position by turning the screw in one direction only. In dividing the circle, burrs are thrown up by the engraving tool on each side of the cut, and reflections from either or both of these appear as fine white lines in the reading microscope. The quartz fibre is made to cover each of these reflections in turn, and the mean taken. Further details of the method are given in the list of indices.

3. *Temperature.*—In order to reduce errors due to change of temperature to a minimum, I have kept a standard thermometer on the prism table, and have never attempted to regulate the temperature by raising or lowering that of the room, without allowing at least 2 hours to elapse before making observations, so as to ensure the prism attaining the same temperature. In most cases times were chosen for observation when the temperature was, without artificial means, found to be that required. The temperature adopted for observation was 15° C. In the very few cases in which observations took place at

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and, substituting for δ, &c., this gives

$$\bar{\mu} = \mu \left\{ 1 + \tfrac{1}{24}(\alpha_1^2 + \alpha_2^2 + \alpha_3^2) \frac{\sin\frac{D}{2} \sin\frac{D+2A}{2}}{\sin^2\frac{A}{2} \cos^2\frac{D+A}{2}} \right\}.$$

Putting in the small term D = 40° and A = 60°, the value of the trigonometrical expression is found to be 3 approximately.

Hence,

$$\bar{\mu} = \mu \{1 + \tfrac{1}{8}(\alpha_1^2 + \alpha_2^2 + \alpha_3^2)\}.$$

If α, &c., be in minutes, since the circular measure of a minute is 1/3420, or 0·00029,

$$\bar{\mu} = \mu \{1 + 0{\cdot}00000001\,(\alpha_1^2 + \alpha_2^2 + \alpha_3^2)\}.$$

For fluorite, $\alpha_1 = 1\tfrac{1}{3}$, $\alpha_2 = -2\tfrac{1}{4}$, $\alpha_3 = \tfrac{1}{2}$, so that

$$\bar{\mu} = \mu \{1 + 0{\cdot}00000007\}.$$

The error is almost unity in the seventh figure, which is quite negligible. For the other prisms it is less.

higher temperatures, recourse was had to the table of temperature refraction coefficients. These coefficients are the mean of a separate series of measurements carried out at temperatures at least 10° C. apart.

4. *Material.*—The material of which the prisms were made is quite normal. The fluorite is from Germany, the quartz from Brazil, the calcite from Iceland.

5. *Standard Wave-lengths.* — Rowland's wave-lengths have been adopted wherever available. As a specimen of the work in fullest detail the determination of the index of line C for fluorite is given. (See Appendix I.)

6. *Curves.*—A very severe method of testing refractive indices, when they are fairly close together, is to calculate the form of a thin lens from two of the substances, so as to be achromatic for two definite wave-lengths, and then focal lengths calculated by the formula

$$(\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \frac{1}{F}$$

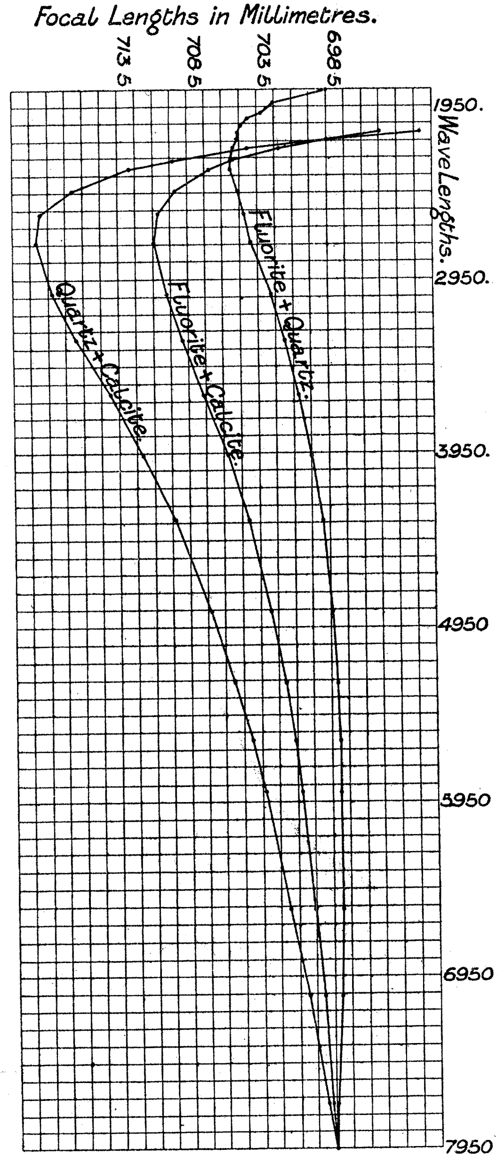
for the different wave-lengths should form the ordinates of a smooth curve. The focal length 6985 mm. was taken as being ten times that required for the lenses of a large spectrometer now just completed. The curves shown (p. 333) are for (A) fluorite and quartz, (B) fluorite and calcite, (C) quartz and calcite. It appears that the focal length of the fluorite and quartz combination is very nearly independent of the wave-length. A list of such focal lengths is given in Appendix II.

7. *Measure of Error.*—An approximate estimate of the error in the deviations may be made as follows :—In reducing a set of observations three means, called "group deviations" (see Appendix I), are taken. Let x, y be values of the two of these which show the greatest difference, then the quantity $a = \frac{x-y}{3}$ may be taken as a measure of the error. There are 118 measurements in the table for

33	of which a = less than $\frac{2}{3}''$;	corresponding variation of index	=	0·0000023
39	,, = ,, $1\frac{1}{3}''$;	,, ,, ,,	=	0·0000034
31	,, = ,, $2\frac{2}{3}''$;	,, ,, ,,	=	0·0000084
15	,, = more than $2\frac{2}{3}''$;	,, ,, ,,	=	,,
1	only where a = as much as $5\frac{1}{2}''$;	,, ,, ,,	=	0·0000150

Optical Correctness of Prisms.—In order to test whether the faces of the quartz and calcite prisms were parallel to the optic axis, each of the angles of the prisms was measured, and from these measurements and the deviations proper to them separate indices were calculated for each of the three angles (a) for the ordinary ray, (b) for the extraordinary ray. Owing for the most part to errors inseparable from measurements

Chromatic Curves for thin Doublets of approximately 7000 millimetres focus.



of angle, the indices are not quite the same at each angle, but if the variations are in the same direction and not much greater for the extraordinary than for the ordinary ray, we may assume that the optic axis is very nearly perpendicular to the principal plane. Taking the three angles as α, β, and γ, the results are as follow (for line D) :—

Quartz.

	Ordinary ray.		Extraordinary ray.
α.	1·5442550	α.	1·5533605
β.	1·5442306	β.	1·5533356
γ.	1·5442765	γ.	1·5533977

Calcite.

	Ordinary ray.		Extraordinary ray.
α.	1·6583705	α.	1 4864145
β.	1·6583381	β.	1·4863865
γ.	1·6583595	γ.	1·4864062

Thus it would seem that the differences in values of the extraordinary indices are not greater than those for the ordinary ray.

As further evidence of accuracy in observation, it may be mentioned that the measurements for line D quartz, ordinary and extraordinary ray recorded in the table, were made on March 4, 1900. On December 27, 1901, the prisms having been set up by mistake for a prism of left-handed quartz, a complete independent series of measurements were again made for both rays before the mistake was discovered. The two resulting indices follow :—

	Ordinary ray.	Extraordinary ray.
March 4, 1900...............	1·5442558	1·5533662
December 27, 1901.........	1·5442558	1·5533673

The difference of the two measurements of the ordinary ray begins in the 8th decimal place and is not shown.

8. *Probable Accuracy of Indices.*—These refractive indices may be considered correct to the 5th decimal place; the 6th is only approximately correct, and the 7th is of little value. But it is believed that in many cases the error does not exceed unity in the 6th place.

9. *Interpolation.*—The question of the relation between refractive index and wave-length has not been gone into, but it may be said that for the ultra-violet Cauchy's formula is not of great value, while for the visual spectrum Nelson's formula is a nearer approximation, and takes little time to work out,* while the graphical interpolation of deviations by squares of reciprocals proposed by Dr. Marshall Watts† is not

* See 'R. M. S. Journal,' April, 1899, Presidential Address.

† 'Index of Spectra,' p. xii.

very inferior to Nelson's, as the indices can soon be calculated, and it can easily be turned into simple arithmetic without the drawing. But the indices here given have all been observed. There has been no attempt to make them fall into line by any kind of correction, such as the use of a freehand curve or by interpolation.

10. *Right and Left Quartz.*—Some doubt having arisen as to the indices of right-handed quartz being the same as those of left-handed, I have had prepared by Mr. Hilger a prism of left-handed quartz of the same dimensions as the prism of right-handed quartz used in the foregoing measurements. I have with this prism determined the indices, ordinary and extraordinary, for the mean D line, and have repeated the corresponding observation with the right-handed prism used in the previous determinations, as before referred to. The results are as follows:—

	Ordinary.	Extraordinary.
March 4, 1900. Right quartz.........	1·5442558	1·5533662
December 27, 1901. Right quartz...	1·5442558	1·5533673
January 9, 1902. Left quartz	1·5442363	1·5533452
Difference	0·0000195	0·0000221

The value given for left quartz and the ordinary ray by Van der Willigen (for wave-length 5895·37, which he names D) is 1·54417.

Finally a rough determination of the specific gravities was made as follows:—

Right quartz.	Left quartz.
2·6495	2·6511

It will thus be seen that while the refractive indices of the right-handed quartz are greater than those of the left-handed, the specific gravity is less.

11. *Results of the Measurements.*—The results of the observations are given in the following table of refractive indices:—

Table of the Refractive Indices of Fluorite, Quartz, and Calcite at 15° C.

Wave-length.	Fluorite.	R. quartz, ordinary.	R. quartz, extraordinary.	Calcite, ordinary.	Calcite, extraordinary.
7950 Rb	1·4306394	‡1·5385126	‡1·5474212	1·6488640	1·4821645
*7682·45 $K_a(A')$	1·4309494	1·5390604	1·5479969	1·6497403	1·4825523
*7065·59 He(B′)	1·4317107	1·5405000	1·5494865	1·6520702	1·4835315
6563·04 $H_a(C)$	1·4325233	1·5419306	1·5509480	1·6543988	1·4845685
*5893·17 Na(D)	1·4338542	1·5442558	1·5533662	1·6583551	1·4863915
Maximum 5607·1 Pb(A) visual effect	1·4345651	1·5454751	1·5546189	1·6604550	1·4873419
*5270·11 Fe(E)	1·4355641	1·5471766	1·5563881	1·6634145	1·4887055
4861·49 $H\beta(F)$	1·4370666	1·5497003	1·5589883	1·6678308	1·4907407
4340·66 $H\gamma(G')$	1·4396260	1·5539758	1·5634058	1·6755174	1·4942419
3961·68 Al	1·4421883	1·5582352	1·5678355	1·6832956	1·4977742
Ultra-violet.					
3610·66 Cd	1·4453386	1·5634731	1·5732431	1·6931606	1·5022352
*3302·85 Zn	1·4490705	1·5697424	1·5797292	1·7051544	1·5074581
3034·21 Sn	1·4533819	1·5769921	1·5872306	1·7195881	1·5136464
2748·68 Cd	1·4596612	1·5875286	1·5981316	1·7415041	1·5226617
2573·12 ,,	1·4647726	1·5962464	1·6071638	1·7605000	1·5301212
*2445·86 Ag	1·4696500	1·6046194	1·6158596	1·7796645	1·5373101
2312·95 Cd	1·4751617	1·6140337	1·6256465	1·8023942	1·5455001

* Wave-lengths for mean intensities of double lines or groups.

† As far as observations go, the co-efficients for other wave-lengths are in the ratio of their refractive indices.

‡ Right-handed quartz; for left-handed quartz, see paper.

2265·13 Cd	1·4775433	1·6181975	1·6299653	1·8130351	1·5491444
2194·4 „	1·4814525	1·6249914	1·6370224	1·8307980	1·5551219
2144·45 „	1·4845693	1·6304659	1·6427036	1·8458240	1·5599225
2098·8 Zn	1·4875705	1·6356982	1·6481440	1·8608110	
*2062·0 „	1·4902594	1·6403981	1·6530369		
2024·2 „	1·4931822	1·6455837	1·6584434		
1988·1 Al	1·4961305	1·6509174	1·6639872		
1933·5 „	1·5012262	1·6600266	1·6734876		
1852·2 „	1·5098894	1·6758953	1·6900687		

Note.—After W.L. 2144·45 Calcite rapidly loses transparency. W.L. 2098·8 was measured with a special thin prism, cut perpendicularly to the axis. With the prisms polished on three faces the axis was parallel to the edge.

Temperature Refraction Coefficients of Wave-length 5893·17 (D)† for 1° C.

−0·00001022	−0·00000519	−0·00000635	0·00000479	0·00001447

The refractive indices of Fluorite and Quartz decrease with rise of temperature, those of Calcite increase.

Method of Observation.

In cutting the goniometric circle a burr is thrown up by the engraving tool on each side of every division. By two small electric lamps suitably arranged behind the reading microscope, either or both burrs are made to appear as fine white lines. With the help of quartz fibres, measurements are made on these, and the mean taken. For every index measurements were made thus: on either side of each of the three angles of the prism, and at three positions of the circle (zero = 0°, 120°, and 240°), thirty-six measurements in all, except in one case (Calcite ordinary ray, wave-length 2098·8), for which a special thin prism which could be polished on two sides only was necessary, and measurements at one angle only could be made. The rays from the collimator were made parallel for each wave-length. These refractive indices may be considered correct to the fifth decimal place, the sixth is only approximately correct, the seventh of little value.

Appendix I.

Details of an Observation as a Specimen.

October 28, 1900. Zeiss Fluor Prism, 3 sides. Temp. 59° F.
W.L. 6563·0 C. (H_α).

	Left of prism.		31	30	36·25	Right of prism.	
	31	31	0·5 ×	left burr	328	29	45·25
α (0°)		30	35·25	mean		29	22·75
		30	21·25 ×	right burr		29	9·25
			31	28	5·125		
	151	28	25·0		88	32	11·25
β (120°)		28	0·0			31	49·75
		27	41·75			31	31·75
			31	30	4·0		
	271	30	16·0 ×		208	30	11·0 ×
γ (240°)		29	54·75			29	46·75
		29	43·5			29	32·0
		A.	31	29	35·125		

			31	28	8·75		
	31	28	39·0		328	32	21·75
β (0°)		28	12·25			31	54·75
		27	52·25			31	31·5
			31	29	56·625		
	151	20	7·5		88	30	19·5
γ (120°)		29	47·0			29	53·75
		29	36·25			29	37·75
			31	30	43·0		
	271	30	58·5 ×		208	29	34·0
α (240°)		30	36·25			29	10·25
		30	25·25 ×			28	55·0
		B.	31	29	36·125		

			31	29	59·0		
	31	30	17·0 ×		328	30	19·25
γ (0°)		29	53·25			29	55·25
		29	39·5			29	41·25
			31	30	37·875		
	151	30	51·5 ×		88	29	40·5
α (120°)		30	30·25			29	14·5
		30	20·0 ×			28	57·25
			31	28	15·25		

	271	28	39·0		208	32	6·5
β (240°)		28	13·75			31	43·25
		27	55·25			31	23·5
		C.	31	29	37·375		

Result.

Dev. = 31° 29′ 36″·208. Ref. Index = 1·4325233.

The above is an exact copy of the original observation for line C (hydrogen α). It will be seen that the readings of the micrometer are arranged in three groups, each of which is complete in itself, since it contains readings at all the angles and at each of the three positions of the circle. The averages for each group, called A, B, and C, are underlined twice, and the final result is the mean of these three averages. The three angles of the prism are called α, β and γ: each of these gives the deviation under measurement three times, that is, once at each of the three positions of the circle (zero = 0°, 120° and 240°). The two burrs thrown up by the engraving tool on each side of every division, already described, give the first readings, and in taking the mean of these a deduction of 1″ for every 1′, measured in the microscope, is made for the optical error of the reading microscope, a careful measurement of the error having come out within a small fraction of this. The microscope was furnished with a fixed and a movable quartz fibre; sometimes the movable fibre was too close to the other to read with accuracy, and then had to be moved off to the next division of the circle (5′ further off centre). In this case I have marked the reading with a ×, so as to ensure attention being called to the matter. After turning the same angle of the prism round to the other side as usual, another mean is obtained from two burr readings in the same way. These two means, one on each side, underlined once, give by the usual method the value for each angle in the groups already described.

Appendix II.

Table giving the Focal Lengths in metres of a Compound Lens of Fluorite and Quartz achromatised for Wave-lengths 7950 and 1852.

Radii.—R = 0·28358015, S = − 0·20801615, R′ = S, S′ = ∞, R, S, R′, S′ refer to the surfaces of the two lenses.

Wave-length.	Focal length.	Wave-length.	Focal length.
7950	1·00000	2749	0·99382
7652 A′	1·00005	2573	0·99314
7065 B′	1·00063	2445	0·99275
6563 C	1·00073	2313	0·99208
5893 D	1·00081	2265	0·99225
5607 A	1·00076	2194	0·99233
5270 E	1·00061	2144	0·99267
4861 F	1·00022	2099	0·99281
4341 G′	0·99945	2062	0·99299
3962	0·99857	2024	0·99356
3611	0·99750	1988	0·99461
3303	0·99654	1933	0·99563
3034	0·99548	1852	1·00000

Graphical representations are given in the curves on p. 333, the achromatised focal length being there 6985 instead of 1000 mm.

Appendix III.

Partial and Proportional Dispersions of Fluorite, Quartz, and Calcite, and their Combinations.

Substance.	C to F ($\delta\mu$).	$\mu_D - 1$.	$\frac{\mu - 1}{\delta\mu} = \nu$.	$\frac{\nu}{\nu'} = N$.	A′ to D.	D to F.	F to G.
Fluorite	0·0045433	0·4338542	97·493		0·0029048	0·0032124	0·0025594
					0·63936	0·70706	0·56333
				1·392	0·02931	−0·00633	−0·01306
Quartz	0·0077697	0·5442558	70·048	..	0·0051954	0·0054445	0·0042755
					0·66867	0·700735	0·55027
				1·438	−0·02731	0·00473	0·02199
Calcite	0·0134320	0·6583551	49·013	..	0·0086148	0·0094757	0·0076866
					0·64136	0·70546	0·57226
				1·989	0·00200	−0·00160	0·00893
Fluorite...............	0·0045433	0·4338542	97·493	..	0·0029048	0·0032124	0·0025594
					0·63936	0·70706	0·56333

Note.—The last three columns give details as to the dispersions between the lines indicated. In each group of three lines connected with a given material the first line gives the differences in refractive indices, the second line the ratio of these differences to the mean dispersion $\delta\mu$, and the third line, in old-faced type, the outstanding secondary dispersion when lenses of the given materials are combined.