"On the Bending of Waves round a Spherical Obstacle." By Lord Rayleigh, F.R.S.

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In the 'Proceedings' for January 21, 1903, Mr. H. M. Macdonald discusses the effect of a reflecting spherical obstacle upon electrical and aerial waves for the case where the radius of the sphere is large compared with the wave-length (λ) of the vibrations. The remarkable success of Marconi in signalling across the Atlantic suggests a more decided bending or diffraction of the waves round the protuberant earth than had been expected, and it imparts a great interest to the theoretical problem. Mr. Macdonald's results, if they can be accepted, certainly explain Marconi's success; but they appear to me to be open to objection.

If C be the source of sound, P a point upon the sphere whose centre is at O, ϕ_1 the velocity-potential at P due to the source (in the absence of the sphere), χ the angle subtended by OC, Mr. Macdonald finds for the actual potential at P,

$$\phi = \phi_1 (1 - \cos \chi) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1),$$

so that there is no true shadow near the surface of the sphere. If C be infinitely distant, and μ denote (as usual) the cosine of the angle between OP and OC,

$$\phi = \phi_1 (1 + \mu) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

That the sound should vanish at the point opposite, and be quadrupled at the point immediately under the source is what would be expected; but that (however large the sphere) the shadow should be so imperfect at, for example, $\mu = -\frac{1}{2}$, is indeed startling.

The first objection that I have to offer is that nothing of this sort is observed in the case of light. The relation of wave-length to diameter of obstacle is about the same in Marconi's phenomenon as when visible light impinges upon a sphere 1 inch (2·54 cm.) in diameter. So far as I am aware no creeping of light into the dark hemisphere through any sensible angle is observed under these conditions even though the sphere is highly polished.*

But I shall doubtless be asked whether I have any complaint against the mathematical argument which leads up to (2).

As in *Theory of Sound*, § 334, the question relates to the ratio between a certain function of c (the radius) and its differential

* It may be remarked that at the centre of the shadow thrown at some distance (say 1 metre) behind, there is a bright spot similar to that seen when a disc is substituted for the sphere. This effect is observed with a magnifying lens. If the eye, situated at the centre of the shadow, be focused upon the sphere, the edge of the obstacle is seen bounded by a very narrow ring of light.

coefficient with respect to c. The function is that which occurs in the representation of a disturbance which travels outwards, and (§ 323) may be denoted by

$$\frac{e^{-i\kappa c}}{c} f_n(i\kappa c) \quad \ldots\ldots\ldots\ldots \quad (3),$$

where

$$\kappa = 2\pi/\lambda,$$

and

$$f_n(i\kappa c) = 1 + \frac{n(n+1)}{2 \,.\, i\kappa c} + \frac{(n-1\ldots(n+2)}{2 \,.\, 4 \,.\, (i\kappa c)^2} + \;.\;.\;.\; \ldots\ldots \quad (4).$$

The differential coefficient of (3) is

$$-\frac{e^{-i\kappa c}}{c^2}\{(1+i\kappa c) f_n(i\kappa c) - i\kappa c f_n'(i\kappa c)\} \ldots\ldots\ldots \quad (5),$$

so that the ratio in question takes the form

$$\frac{-c f_n(i\kappa c)}{(1+i\kappa c) f_n(i\kappa c) - i\kappa c f_n'(i\kappa c)} \quad \ldots\ldots\ldots \quad (6).$$

In these expressions n is the order of the Legendre's function $P_n(\mu)$ which occurs in the series representative of the velocity-potential.

When κc is very great, the ratio expressed in (6) may assume a simplified form. From (4) we see that, if n be finite,

$$f_n(i\kappa c) = 1, \qquad i\kappa c f_n'(i\kappa c) = 0,$$

ultimately, so that

$$(6) = -\frac{1}{i\kappa}, \quad \ldots\ldots\ldots\ldots \quad (7),$$

independent of n.

This is the foundation of the simple result reached by Mr. Macdonald. Its validity depends, therefore, upon the applicability of (7) to all values of n that need to be regarded. If when κc is infinite, only finite values of n are important, (7) is sufficiently established; but (§ 328) it appears that under these conditions the most important terms are of infinite order. I think it will be found that for the most important terms n is approximately equal to κc, and that accordingly (7) is not available. In any case it could not be relied upon without a further examination.

In *Theory of Sound*, § 328, the problem is treated for the case where κc is small, and the calculation is pushed as far as $\kappa c = 2$. The results indicate no definite shadow. I have commenced a calculation for $\kappa c = 10$, about the highest value for which the method is practicable. But it is doubtful whether even this value is high enough to throw light upon what happens when κc is really large.