"Some Uses of Cylindrical Lens-Systems, including Rotation of Images."

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It was, I believe, first shown by the late Sir George Stokes that if two similar cylindrical lenses are placed in contact, with their axes of curvature at right angles, the combination acts as a spherical lens, and that if the axes are not at right angles the system is equivalent to a cylindrical and a spherical lens combined, the spherical element disappearing when the axes are parallel.

I have recently had occasion to employ a modification of this arrangement which has proved of some practical utility as well as theoretical interest.

When, as in Sir George Stokes's experiment, the two lenses are placed in contact, the combination acts as a spherical lens in every respect, that is to say, the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ holds in regard to it.

But when the two lenses are not in contact there is only one pair of conjugate points at which a real object will give a real image. Reference to the ordinary focometer formula will show that this must be the case. Let l be the distance between an object and a screen, and let a convex lens of focal length $f < \frac{1}{4}l$ be moved along the line joining them. Then there must be two, and only two, positions of the lens at which it will form images on the screen, one at a distance u from the object and v from the screen, and the other at a distance v from the object and u from the screen where

$$u+v = l, \quad u-v = a, \quad \text{and} \quad \frac{l^2-a^2}{4l} =$$

In either of these two positions a cylindrical lens of equal power will, if its axis be vertical, form sharp images of vertical lines, or of horizontal lines if its axis be horizontal. If therefore we place one cylindrical lens with its axis vertical at a distance u from the object and a second of equal power with its axis of curvature horizontal at a distance u from the screen, there will be produced on the screen a sharp image magnified vertically v/u times, and horizontally u/v times.

This method may be employed for comparing by photography curves plotted to different scales, or for increasing or diminishing the ordinates of a curve or record the scale of which is unsuitable. Figs. 1 and 2 are an example of this latter use.

Fig. 1 is a record taken with the capillary electrometer of the discharge of the electrical organ of Malapterurus, the two sides of the

organ being connected with the outer coatings of a pair of condensers, the inner coatings of which were connected with the electrometer.

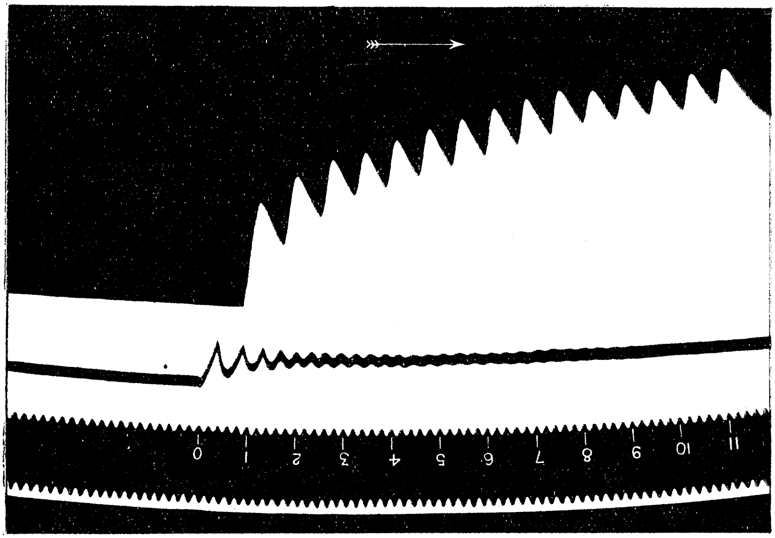

FIG. 1.

A full description of the experiment is given in the papers by Gotch and myself, from which this illustration is taken.*

In order to find the E.M.F. indicated by such a record it is necessary to measure the subtangent—or, in this case, the polar subnormal to the curve at various points according to the method described in my papers on the "Capillary Electrometer."† The method is, however, inapplicable if the angles are too steep.

It occurred to me that if the ordinates could be diminished and the abscissæ magnified by photography the angles might be brought within the measurable range. Accordingly I fixed the original of fig. 1 at 56·25 cm. from a photographic plate, with a cylindrical lens of +12·5 cm. with its axis horizontal at a distance of 17·5 cm., and a second with its axis vertical at a distance of 43·75 cm. The result is shown in fig. 2. The definition is remarkably good in the negative,

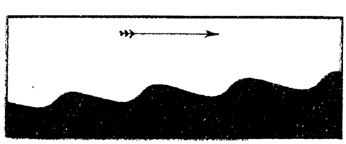

FIG. 2.

* 'Phil. Trans.,' B, vol. 187, p. 347; 'Proc. Roy. Soc.,' vol. 65, p. 434.

† 'Phil. Trans.,' A, vol. 183, p. 81; 'Proc. Roy. Soc.,' vols. 48, 59, 60, p. 388, and vol. 70, p. 175.

but the field of view is small. The abscissæ are magnified two and a half times and the ordinates diminished in the same proportion, so that the total effect is the same as if the original photograph had been taken on a plate moving 25/4 times as fast, and the resulting curve can be easily analysed.

It should, however, be noted that fig. 1, being a curve with polar co-ordinates, is not properly adapted for this process, which requires, to be accurate, curves with rectangular co-ordinates. It serves, nevertheless, to illustrate the method, and as the portion photographed occupies not more than an angle of 2° of arc, the error involved is hardly noticeable although great enough to render it not worth while to spend time over a minute analysis of the modified curve.

It is not necessary that both lenses should be of the same focal length. If they are, the image is magnified as much in one direction as it is diminished in the other. If it is desired to preserve the dimensions in one direction unaltered—as, for instance, in comparing the time relations of two electrical responses of different intensity—*e.g.*, nerve under the influence of CO_2, and in a normal condition—one lens may be put midway between the object and the screen.

If m be the magnification required in the other direction, the focal length necessary for the other cylindrical lens is $f = \frac{m}{1+m} \cdot l$ and its distance from the object is $\frac{l}{1+m}$.

In practice it is more convenient to employ a pair of lenses of rather more than the required power when close together, and to adjust the distance between them until the right magnification is obtained.

The image thus modified by the cylindrical system is projected on to the curve with which the comparison is to be made, and examined with a lens.

The principal difficulty is the smallness of the field of view, which is elliptical in shape, the major axis being limited by the diameter of the lenses and the ratio between the axes being as $1 : m^2$. Unfortunately the use of lenses of larger aperture introduces errors of chromatic and spherical aberration which spoil the definition.

Owing to the fact that each cylindrical lens has to be separately focussed, and that no image is formed until both are correctly adjusted, not only as to distance but also as to the direction of the axes of curvature, focussing is a matter of some difficulty. I have found the following method answer very well:—

Two lines, one vertical and the other horizontal, are ruled, across the middle of a glass plate—a spare negative answers very well, and the lines can be ruled with a needle on the film. This is placed in the object holder. The first cylindrical lens is inserted in its cell and the horizontal line focussed with it sharply on the screen. The first lens

being removed, the second is inserted in the other cell and adjusted to focus the vertical line.

The first lens is then replaced in position and rotated till both lines are sharply focussed. When three lenses are used, the two employed to obtain magnification must be contained in a sliding tube furnished with a pin working in a slot to prevent rotation.

In the preceding pages the axes of curvature of the two components of the cylindrical lens-systems are, in all cases, at right angles to each other. Equally interesting properties are possessed by those systems in which they are parallel to each other.

CASE I.—*Two equal convex cylindrical lenses set with their axes of curvature parallel, and at such a distance apart that their principal foci coincide as in a telescope.*

A clear and undistorted view of distant objects is obtained, but on rotating the object the image appears to rotate with equal angular velocity in the opposite direction, and on rotating the tube containing the lenses, the image appears to rotate with twice the angular velocity in the same direction.

The reason is obvious. Suppose the axes of curvature are vertical, then there is no vertical deviation of the image-forming rays, and so far as its vertical components are concerned the virtual image coincides with the real object. In the horizontal plane there is deviation. The rays from the cylindrical objective cross as in a telescope before reaching the cylindrical eyepiece. If, therefore, we focus the eyepiece so that the final virtual image is at the same distance from the eye as the real object, a clearly defined image will be produced, erect, but enantiomorphic, as if reflected in a plane mirror. If the object is viewed through two cylindrical telescopes in tandem, then if either is rotated, the image rotates in the opposite direction, but if both are rotated together, the image remains stationary as in the case of two erecting prisms in tandem.

Two causes may disturb the sharpness of the definition, namely, deviation from exact parallelism of the axis, and error in adjusting the distances between the lenses, so that the horizontal components of the virtual image are not focussed at the same distance as the vertical.

If the two lenses are not of equal focal length the resulting images are clearly defined, but not symmetrical, being magnified or diminished in one direction and of natural size in the other.

CASE II.—*Two cylindrical lenses with their axes of curvature parallel, the distance between them being greater than the sum of their focal lengths.*

Such a system acts as a compound microscope, giving well defined images of objects situated at a *certain distance* from the objective.

The same phenomena of rotation of the image on rotating the lens system are observable in this case as in the cylindrical telescope, but with this difference: the images, though sharply defined, are not symmetrical, the object being magnified at right angles to the axis of curvature, and not magnified parallel to it.

The focal distance at which alone good definition is obtainable is that at which the virtual image is the same distance from the eye as the object. But to an astigmatic eye there are two distances at which the instrument gives perfect definition, the direction of the axis of curvature coinciding with the astigmatic axis of the eye in one case and being at right angles to it in the other.

The cylindrical telescope cannot by altering the focussing be made to project an image on a screen like an ordinary telescope. To do that we must employ two cylindrical telescopes with their axes of curvature at right angles. They need not both be of the same power, and, curiously enough, one may be situated inside the other without interfering with its action.

Polarised Light.

Another peculiarity of the cylindrical telescope is rather remarkable and extremely valuable. As would be expected, rotation of the tube of a cylindrical telescope, though it rotates the image, is absolutely without effect as regards the polarisation of the ray. There is, so far as I know, no other means of rotating an image without altering it in this respect, the ordinary erecting prism, whether silvered or unsilvered, introducing a difference of phase. I have found the cylindrical telescope extremely useful in some experiments where this was important.

Note added March 12.

If the axis of a telescope of which the magnifying power is m is moved sideways through an angle θ, the displacement of the image is $(1 \pm m)\theta$, taking the upper sign if the image is inverted, and the lower sign if it is erect.

In the telescope formed of two equal cylindrical lenses, parallel to the axis of curvature $m = -1$, and at right angles to it $m = +1$. Accordingly, if the axis of such a telescope is moved through an angle θ in the plane of the axis of curvature, the displacement of the image is zero. If it is moved in a plane at right angles to this the displacement is 2θ. And if the plane of movement makes an angle of 45° with the axis of curvature, the image appears to move at right angles to it. The effect of rotating the telescope about its axis while moving it in azimuth is very striking.

A cylindrical telescope may be made with one lens as follows:—A

plane mirror is set at right angles to the optic axis in the principal focus of a cylindrical lens. Between the lens and the eye a plate of unsilvered glass is placed at such an angle as to reflect the rays from some distant object through the lens on to the mirror, whence they are reflected once more through the lens into the eye. The unsilvered reflection is not necessary if the eye is held at some considerable distance from the lens. In such an arrangement the image is not enantiomorphic, the effect of the cylindrical lens-system being neutralised by that of the mirror, but the image rotates when the lens is rotated exactly as in an ordinary cylindrical telescope, and the plane of polarisation is not affected by the rotation.

"On the Effect of a Magnetic Field on the Rate of Subsidence of Torsional Oscillations in Wires of Nickel and Iron, and the Changes Produced by Drawing and Annealing." By Professor ANDREW GRAY, F.R.S., and ALEXANDER WOOD, B.Sc. Received March 12,—Read March 17, 1904.

On May 15th, 1902, we communicated to the Royal Society a paper entitled "On the Effect of a Longitudinal Magnetic Field on the Internal Viscosity of Nickel and Iron, as shown by Change of the Rate of Subsidence of Torsional Oscillations." We described in that paper the results of experiments on the rates of subsidence of torsional oscillations in nickel and iron wires in fields of different strengths, and showed that the effect of the field, or, more properly, of the magnetisation of the wire, is to increase the rate of subsidence in nickel and to diminish it in iron. In nickel, it was pointed out, this effect rose to a maximum at a certain field, from 100—180 C.G.S., according to the initial amplitude, and thereafter diminished as the field was increased; while in iron the effect was in the main all produced at a field of about 160 C.G.S. or rather less, and increased only slightly with further increase of field intensity.

The experiments described in the present paper are referred to at the end of the former one as in progress, and some account of their results is given; and we propose now to describe them a little more in detail.

Experiments on the Effect of Drawing Down and of Annealing a Nickel Wire.—A piece of the nickel wire formerly experimented on was tested for subsidence in the manner already described, and then drawn down, by being passed through a draw plate, from the diameter 1·4 mm. to 0·775 mm. The results are illustrated by fig. 1. Take first curve I of that figure. It has for ordinates the differences between