"An Experiment Illustrating Harmonic Undertones." By

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An Experiment Illustrating Harmonic Undertones.

The object of this note is to describe a phenomenon of which, as far as I have been able to learn, no published account has hitherto been given, although perhaps its extreme simplicity renders it unlikely that it has altogether escaped detection. If a vibrating tuning-fork is pressed against a light object, such as a piece of paper or a stretched string, this object will in general follow the vibrations of the fork until they die away, remaining in contact with the fork all the time. This method of using a tuning-fork is mentioned in Lord Rayleigh's "Theory of Sound," § 133. If, however, a piece of paper is touched lightly by the vibrating fork, the elastic force opposing displacement of the paper may be so small that it does not remain in contact with the fork, but is driven away by the tap which it receives. The paper may return in time to be struck by the fork before the latter has executed a complete vibration, and the process being repeated again and again the paper will vibrate with the same period as the fork, but with a motion which is not simple-harmonic, on account of the irregularity caused by the blows. The result of this is that the paper emits a note in which the harmonic overtones are of considerable importance, and which, therefore, resembles the note of a bowed violin string rather than the almost pure tone of the tuning-fork. This resemblance is easily perceived, as is also the change to the approximately pure tone which takes place when contact ceases to be broken, on account of increase of pressure or falling off in the amplitude of vibration of the fork. (The vibration of the air in the neighbourhood of the fork has also an effect on the motion of the paper, but this effect is probably so small in comparison with that of the blows that it may here be omitted.)

If the paper does not come within striking range of the fork again until the latter has nearly had time to execute two complete vibrations, the note given by the paper will be an octave below that of the fork, since the vibrations of the paper have now a period which is double that of the fork. Thus, supposing the pitch of the fork to be c'', the paper will give the note c'. If the fork strikes the paper at every third vibration the paper will give f, a twelfth below the note of the fork. Similarly contact at every four, five, six, or seven vibrations will give c, A♭, F, or a note a little sharper than D, the ratio of

their frequencies being 64 : 63. We can thus obtain from the paper the series of harmonic undertones

$$c', f, c, \mathrm{A}\ , \mathrm{F}, \mathrm{D}+, \mathrm{C}, \mathrm{B}_{,}\flat, \mathrm{A}_{,}\flat, \text{ etc.}$$

It is easy to obtain about the first ten of these when a small slip of paper is used and is held close to the ear. There is some difficulty in making the lower notes audible on a large scale without depriving the experiment of the simplicity which is probably its chief merit; it is perhaps best to use a large sheet of paper suspended in a vertical plane with one edge pasted to a vertical edge of an empty wooden box which may reinforce the sound. The tuning-fork can be held in the hand or clamped so as to touch the paper near its edge; the undertone usually given will vary with the position of the point of contact. The effect might be prolonged for any time by using an electrically maintained fork.

Some further information as to the behaviour of a light object under the influence of a series of periodic blows was obtained by means of a large steel tuning-fork with prongs about a foot long, which was clamped in such a position that one of its ends, vibrating horizontally, struck the upper end of a small vertical card (usually an ordinary visiting card), the lower end of which was fixed, so that the card behaved somewhat as a clamped-free rod of considerable width in comparison with its length. The average pressure and the position of the point of contact could be varied at will. The fundamental pitch of the fork was itself so low that harmonic undertones, produced by the card in the way described above, were practically inaudible; the nature of the vibration was, however, examined by looking through a lens at the upper edge of the card against a dark background. The edge appeared to be drawn out into a continuously shaded band, in which occurred at intervals somewhat abrupt white lines, indicating positions at which the card was nearly or quite stationary. When contact was being broken several of these lines were usually visible, and it was possible to get them to remain steady, though sometimes they flickered rapidly. The line at the end of the band nearest the fork was usually very faint, often practically invisible, which points to the suddenness with which the card is driven away from the fork when they come into contact.

The presence of these white lines shows that the card executes vibrations corresponding to more than one of its free modes, and that the resultant vibration differs considerably from the simple harmonic form. (This agrees with the audible presence of harmonics in the experiment first described, where a sheet of paper is struck by an ordinary tuning-fork.)

As the vibrations of the large fork diminish in amplitude the white lines become fewer, until generally only two or one remain in the

interior of the band, in addition to the line at the end of the band away from the fork. If the amplitude of the fork dies down still further or the pressure between the card and the fork is increased, the lines disappear from the interior of the band and that at the end next to the fork becomes as strongly marked as the line at the other end. This, of course, indicates that contact has ceased to be broken, and the only audible note given by the card is now the fundamental note of the fork.

"Further Note on the Remains of *Elephas cypriotes*, Bate, from a Cave-Deposit in Cyprus." By DOROTHY M. A. BATE. Communicated by Dr. HENRY WOODWARD, F.R.S. Received April 18,—Read June 9, 1904.

(Abstract.)

This paper is a continuation of one *already published*,* "On the Discovery of a Pigmy Elephant in the Pleistocene of Cyprus," and enters into a detailed description of the teeth of this small proboscidean, whose remains are now in the British Museum of Natural History.

The collection includes incisors, milk molars, and permanent molars. Several of the latter still retain their position in the jaws and, in some instances, the teeth of both sides of the same individual were found.

The permanent incisor tusks of two forms, presumably belonging to males and females, were found. They differ from the same teeth of the Maltese dwarf elephants in being considerably compressed laterally. The largest specimen measures 29·7 cm. along the outside of the curve, with a maximum diameter of 3·7 cm.

Of the upper cheek teeth the third and fourth of the milk series as well as the three permanent molars are described in detail. There was a small third milk molar (mm. 2) implanted by a single root, but no specimen was collected. Of the lower series the third and fourth milk molars and the three permanent teeth were represented by numerous examples and are fully described. An almost entire left ramus of one young individual and the symphysial portion of another are also described. The only limb bone obtained was the distal portion of a femur.

A corrected ridge formula for the molars of *E. cypriotes* is furnished, which, exclusive of talons, will stand as follows:—

$$\div,\ \frac{5}{5},\ \frac{7-8}{7-8},\ \frac{7-8}{7-8},\ \frac{8-9}{8-9},\ \frac{11-12}{11-12}.$$

* Read before the Royal Society, May 7, 1903.