"Memoir on the Theory of the Partitions of Numbers. Part III." By Major P. A. MacMahon, Sc.D., F.R.S. Received November 21,—Read December 8, 1904.

(Abstract.)

In this communication a "general magic square" is defined as consisting of n^2 numbers arranged in a square in such wise that each row, column, and diagonal contains a partition of the same number.

Such squares can be added together, by addition of corresponding numbers, without losing the magic property, and we can thus speak of a linear function of squares of the same order, the coefficients being integers. The squares can, in fact, be regarded as numerical magnitudes, and can be taken as the elements of a linear algebra. Since, moreover, arithmetical addition can be made to depend upon algebraical multiplication, the properties of the magnitudes can be investigated by means of a non-linear algebra.

The properties of a magic square can be exhibited by means of a system of homogeneous linear diophantine equations, so that it immediately follows that there is syzygetic theory of such formations.

The method of procedure is set forth and worked out in detail for the third order.

In Section 10 the general question of enumeration associated with a given sum is considered, and some particular results obtained.

The methods are applicable not only to general magic squares as herein defined, but to all cases of forms in "Arithmetic of Position," which retain their properties after addition of corresponding elements.