# *The Growth of the Body in Man.—The Relationship between the Body-weight and the Body-length (Stem-length).*

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A number of investigations carried out in recent years by Prof. Dreyer and myself have indicated the existence of several previously unknown relationships which hold throughout a given species of animals between functions of the body-weight and various other ascertained measurements.*

In connection with work of a similar character on which we are engaged, I undertook some years ago the collection and examination of data bearing on the growth of Man. This work is still incomplete, but certain results of interest have already emerged.

The present communication deals with the relationship between the length of the human body and its weight during the period of growth. By the term "length" of the body is here meant the length of the stem of the body constituted by the head, neck and trunk; that is to say the distance from the top of the head to the line joining the ischial tuberosities, or stem-length as it may be termed.

This measurement corresponds to the body-length of animals and was chosen as the first object for investigation in order that any results which were obtained in Man might be comparable with those obtained in other species.

## *Method of Measurement.*

*Length.*—The length is measured by seating the subject on the floor or on a low table (not a chair) with the back against the wall. Care is taken to see that the sacrum is in contact with the wall, and the legs somewhat drawn up so that the individual sits fairly upon his ischial tuberosities. Under these conditions the height of the top of the head gives a true measurement of the length of the body, and one which is constant and incapable of variation by the subject.

If a chair or other form of seat be employed in taking this measurement

* Dreyer, Georges, and Walker, E. W. Ainley, "The Determination of the Minimal Lethal Dose of various Toxic Substances and its Relationship to the Body-weight in Warm-blooded Animals, together with Considerations bearing on the Dosage of Drugs," 'Roy. Soc. Proc.,' B, vol. 87, p. 319 (1914), and references to literature contained therein.

the individual can by "sitting low" or "sitting high" produce at will a variation of as much as 3 or more per cent. But, since a subject conscious that he is being measured for height tends naturally to produce a full measurement, it will be found that he intentionally "sits up," straightening the spine, tilting the pelvis forward, and rests on the contracted muscles of the thighs and buttocks instead of on his ischial tuberosities. The apparent length—"sitting height," as it has been termed—is thus increased by between 2 and 3 per cent. above the measurement of length taken in the manner already described. Accordingly measurements taken on a seat require to be corrected down appropriately before they can be treated as comparable with the measurement of body-length in infants or animals.

In infants too young to sit the measurement of length is carried out as follows. Two pairs of hands are required. A table is brought up against the wall and the infant is laid upon the table on its back so that the head carefully held in position just touches the wall. The pelvis is held down upon the table, the thighs flexed on the trunk, and a vertical flat piece of board is brought up against the nates. The distance between the board and the wall gives the length of the body.

*Weight.*—The weight of the body is, of course, the weight without clothes.

### *Data and Calculations.*

The data employed in the present investigation consist in part of measurements made by myself; and in part of measurements kindly obtained for me by Mr. G. Haynes at the Oxford Preparatory School; by Mr. Carter at the New College (Oxford) Choir School; and by Miss Poulton at Malvern College for Girls. To each of them I am very greatly indebted for the care and accuracy with which my directions for measurement were carried out. All the measurements in infants and very young children, as well as some in older children, were made by myself, Mr. Haynes supplied the data for 94 boys, Mr. Carter those for 19 boys, and Miss Poulton those for 56 girls.

The subjects were all healthy normal individuals, well nourished and living under favourable conditions. Their average weight and height for each year of age are above those usually given in tables for the general population.

In addition to the data already mentioned I had at my disposal, by the kindness of Dr. E. H. J. Schuster, measurements made by him on 1500 undergraduates at Oxford.

These data have formed the basis of a series of calculations whose result has been to show that the length of the body (stem-length) in man can

correctly be expressed as a function of the body-weight, and conforms to the formula

$$l = kW^n,$$

where $l$ is the length of the body (stem-length) in millimetres, W the weight (without clothes) in grammes, $k$ a constant, and $n$ a power of the approximate value $\frac{1}{3}$. The evidence on which this conclusion is based will now be presented.

*Males.*

Table I (not printed, but preserved for reference in the archives of the Royal Society) contains a full record of the data for boys, together with the calculated value of the length constant $k$ for each individual, deduced from the formula $k = W^n/l$, where $n$ has the value 0·33 as determined in Table II.

In Table II the boys are grouped according to weight in twenty groups. The average body-weight and average body-length for each group is set out, and the figures in the various columns are calculated for each group from the average body-weight and body-length of the group. The body-weights of the groups cover a range in weight from 8168 grm. to 51,480 grm. and show a more than six-fold increase from the lightest group to the heaviest.

In the first instance the "best $n$" for these groups in the formula $l = kW^n$ was ascertained graphically to lie in the neighbourhood of 0·3. The precise values of $n$ and $k$ were then determined by trial from the formula $\log k = \log l - n \log W$. The "best $n$" was thus found to have the value 0·33 while the value 0·32 for $n$ is nearly as good.

The values of $k$ for the groups are shown in the columns for length constant calculated and are seen to be free from periodicity. They give an average value for $k$ of 23·23 when $n$ is 0·33, and a value of 25·73 when $n$ is 0·32.

Substituting these values of $n$ and $k$ in the formula $l = kW^n$ the theoretical value of $l$ is calculated for each group in the appropriate column. The observed values of $l$ are in good agreement with these calculated values and show an average deviation from them of only 1·32 per cent.

If account be taken of the number of individuals in each group the average deviation becomes 1·10 per cent. when $n$ is 0·33, and 1·14 per cent. when $n$ is 0·32. The mean deviation calculated by the method of least squares is, under the same circumstances, 1·48 per cent. when the best $n$ (0·33) is used, and 1·50 per cent. when $n$ is given the value 0·32.

It follows, therefore, that the stem-length of boys conforms to the formula

Table II.—Boys Grouped: Data and Calculations.

| No. of group. | No. of individuals in group. | Average body weight, W. | Average body-length (stem-length) observed, $l$. | Body-length as a percentage of body-weight. | Length constant calculated. $k = W^n/l$. $n = 0 \cdot 32$. | Length constant calculated. $k = W^n/l$. $n = 0 \cdot 33$. | Body-length calculated. $l = kW^n$. $n = 0 \cdot 32$. $k = 25 \cdot 73$. | Difference between body-length calculated and observed. $n = 0 \cdot 32$. $k = 25 \cdot 73$. | Body-length calculated. $l = kW^n$. $n = 0 \cdot 33$. $k = 23 \cdot 23$. | Difference between body-length calculated and observed. $n = 0 \cdot 33$. $k = 23 \cdot 23$. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | grm. | mm. | | | | mm. | per cent. | mm. | per cent. |
| 1 | 1 | 51,480 | 800 ·0 | 1 ·55 | 24 ·85 | 22 ·29 | 828 ·1 | 3 ·39 | 833 ·7 | 4 ·04 |
| 2 | 7 | 45,340 | 779 ·6 | 1 ·72 | 25 ·22 | 22 ·66 | 795 ·2 | 1 ·96 | 799 ·3 | 2 ·46 |
| 3 | 5 | 42,690 | 775 ·8 | 1 ·82 | 25 ·59 | 23 ·00 | 780 ·0 | 0 ·54 | 783 ·6 | 1 ·00 |
| 4 | 10 | 40,890 | 773 ·9 | 1 ·89 | 25 ·88 | 23 ·27 | 769 ·3 | 0 ·60 | 772 ·7 | 0 ·16 |
| 5 | 4 | 39,520 | 764 ·5 | 1 ·93 | 25 ·84 | 23 ·26 | 761 ·0 | 0 ·46 | 763 ·8 | 0 ·09 |
| 6 | 7 | 37,620 | 743 ·0 | 1 ·98 | 25 ·52 | 22 ·97 | 749 ·1 | 0 ·81 | 751 ·6 | 1 ·14 |
| 7 | 8 | 36,200 | 741 ·8 | 2 ·05 | 25 ·79 | 23 ·22 | 739 ·9 | 0 ·26 | 742 ·2 | 0 ·05 |
| 8 | 14 | 35,220 | 738 ·6 | 2 ·10 | 25 ·92 | 23 ·34 | 733 ·3 | 0 ·72 | 735 ·3 | 0 ·45 |
| 9 | 13 | 34,010 | 732 ·0 | 2 ·15 | 25 ·96 | 23 ·40 | 725 ·2 | 0 ·94 | 726 ·9 | 0 ·70 |
| 10 | 10 | 32,910 | 729 ·5 | 2 ·22 | 26 ·15 | 23 ·57 | 717 ·6 | 1 ·66 | 719 ·1 | 1 ·45 |
| 11 | 10 | 31,850 | 725 ·8 | 2 ·28 | 26 ·29 | 23 ·70 | 710 ·3 | 2 ·18 | 711 ·4 | 2 ·02 |
| 12 | 11 | 31,030 | 705 ·1 | 2 ·27 | 25 ·75 | 23 ·23 | 704 ·4 | 0 ·10 | 705 ·3 | 0 ·03 |
| 13 | 10 | 28,150 | 687 ·3 | 2 ·44 | 25 ·90 | 23 ·38 | 682 ·6 | 0 ·69 | 682 ·9 | 0 ·64 |
| 14 | 10 | 25,740 | 675 ·1 | 2 ·62 | 26 ·18 | 23 ·65 | 663 ·4 | 1 ·76 | 663 ·1 | 1 ·81 |
| 15 | 6 | 22,360 | 656 ·7 | 2 ·94 | 26 ·65 | 24 ·11 | 634 ·2 | 3 ·55 | 633 ·0 | 3 ·74 |
| 16 | 3 | 20,040 | 607 ·0 | 3 ·03 | 25 ·51 | 23 ·15 | 612 ·4 | 0 ·88 | 610 ·5 | 0 ·57 |
| 17 | 6 | 17,960 | 591 ·3 | 3 ·29 | 25 ·72 | 23 ·32 | 591 ·3 | 0 ·00 | 588 ·9 | 0 ·41 |
| 18 | 3 | 14,670 | 540 ·7 | 3 ·69 | 25 ·27 | 22 ·97 | 550 ·4 | 1 ·76 | 547 ·0 | 1 ·15 |
| 19 | 4 | 12,250 | 502 ·8 | 4 ·10 | 24 ·73 | 22 ·51 | 523 ·1 | 3 ·88 | 519 ·0 | 3 ·12 |
| 20 | 5 | 8,168 | 460 ·4 | 5 ·64 | 25 ·78 | 23 ·56 | 459 ·5 | 0 ·20 | 450 ·0 | 1 ·41 |
| | 147 | Average .............................. | | | 25 ·73 | 23 ·23 | — | 1 ·32 (1 ·317) | — | 1 ·32 (1 ·322) |
| | | Average, taking into account number of individuals in each group ........... | | | | | | 1 ·14 | — | 1 ·10 |
| | | Mean deviation, taking into account number of individuals in each group ... | | | | | | 1 ·50 | — | 1 ·48 |

$l = kW^n$, where $n$ has the value 0·33 and the value of $k$ for the grouped boys is 23·23.

For the individual boys the average value of $k$ is 23·33. Its greatest value in the series (25·27) exceeds the average value by 8·32 per cent., and its least value (21·08) falls below the average value by 9·69 per cent.

Using the average value of $k$ (23·33) and the value 0·33 for $n$, the theoretical body-length of each individual has been calculated from the body-weight by means of the formula

$$l = 23{\cdot}33\ W^{0{\cdot}33}.$$

From consideration of space the figures have not been tabulated, but they show an average percentage difference between the calculated body-length and that actually observed for the individual boys of only 2·63 per cent., and a mean deviation calculated by the method of least squares of 3·378 per cent.

Accordingly it may be taken that the formula holds very satisfactorily for boys. In these boys, however, the ages do not exceed 15 years, nor the weights 51,000 grm. (8 stone 1 lb.). Hence before it can be assumed that the formula holds up to adult age some further evidence is desirable. This may be found in a study of the data provided by Dr. Schuster's measurements of undergraduates, along with certain measurements of undergraduates made by myself.

Dr. Schuster's data consist of the measurements of two series of undergraduates during their first year at the University, a first series of 1000 individuals and a second series of 500. They are presented in Tables III and IV as grouped by myself.

In dealing with these figures for grouped undergraduates it was necessary at the outset to apply *two corrections.* In the first place Schuster's measurement of body-length is the "sitting height" as measured on a fixed seat. But from observations which I made on a series of individuals measured for the purpose in a similar manner, and also in the manner described in this paper, it appeared that the sitting height by the former method is always greater by from 2 to 3 per cent. than the measurement of length on which the present investigation is based. Between the limits 2 and 3 per cent. the precise value of the excess measurement depends very much on how the individual seats himself, and on the degree to which he lifts himself on his thigh muscles as explained in the section on method of measurement.

Accordingly I have taken the mean of these limits (*i.e.* 2·5 per cent.) as the amount to be deducted from the average sitting heights to obtain the true value of the average body-lengths for the groups. The second correction

Table III.—Undergraduates—First Series—Data, Corrections, and Calculations.

| Group. | No. in group. | Average body-weight partly clothed, $w$. | Average sitting height, $\lambda$. | Weight of part clothes calculated. $c = kw^n$, $n = 0{\cdot}29$, $k = 69{\cdot}63$. $c$. | Body weight corrected, W. | Body-length corrected. $l = 975\lambda/1000$ (stem-length) $l$. | Body-length as a percentage of body-weight. | Length constant calculated. $k = W^n/l$, $n = 0{\cdot}33$. | Length constant calculated using "best $n$." $n = 0{\cdot}20$. | Body-length calculated. $l = kW^n$. $n = 0{\cdot}20$. $k = 98{\cdot}22$. | Difference between body-length calculated and observed. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | grm. | mm. | grm. | grm. | mm. | | | | mm. | per cent. |
| 1 | 7 | 92,080 | 975 | 1916 | 90,160 | 951 | 1·05 | 22·03 | 97·07 | 962 | 1·14 |
| 2 | 40 | 84,820 | 971 | 1871 | 82,950 | 947 | 1·14 | 22·55 | 98·28 | 946 | 0·11 |
| 3 | 55 | 80,290 | 961 | 1838 | 78,450 | 937 | 1·19 | 22·73 | 98·35 | 936 | 0·11 |
| 4 | 77 | 77,570 | 955 | 1827 | 75,740 | 931 | 1·23 | 22·84 | 98·40 | 929 | 0·22 |
| 5 | 119 | 74,390 | 951 | 1801 | 72,590 | 927 | 1·28 | 23·07 | 98·83 | 921 | 0·65 |
| 6 | 161 | 71,220 | 939 | 1778 | 69,440 | 916 | 1·32 | 23·14 | 98·54 | 913 | 0·33 |
| 7 | 162 | 68,040 | 933 | 1755 | 66,280 | 910 | 1·37 | 23·33 | 98·99 | 905 | 0·55 |
| 8 | 151 | 64,410 | 925 | 1728 | 62,680 | 892 | 1·42 | 23·30 | 97·95 | 895 | 0·34 |
| 9 | 111 | 61,690 | 913 | 1706 | 59,980 | 890 | 1·48 | 23·59 | 98·58 | 887 | 0·34 |
| 10 | 65 | 58,970 | 900 | 1684 | 57,290 | 878 | 1·53 | 23·62 | 98·15 | 879 | 0·11 |
| 11 | 34 | 55,790 | 884 | 1657 | 54,130 | 862 | 1·59 | 23·63 | 97·45 | 869 | 0·81 |
| 12 | 18 | 51,710 | 876 | 1621 | 50,090 | 854 | 1·70 | 24·02 | 98·06 | 856 | 0·23 |
| | 1000 | | | | Average ............................ | | | (23·15) | 98·22 | — | 0·41 |
| | | | | | Average, taking into account number of individuals in each group ........... | | | | | | 0·38 |
| Average of groups .................................... | | | | | 68,320 | 908 | | | | | |
| Average of individuals ................................ | | | | | 67,270 | 908 | | | | | |

Table IV.—Undergraduates—Second Series—Data, Corrections, and Calculations.

| Group. | No. in group. | Average body-weight partly clothed, $w$. | Average sitting height, $\lambda$. | Weight of part clothes calculated. $c = kw^n$. $n = 0 \cdot 29$. $k = 69 \cdot 63$. $c$. | Body-weight corrected, W. | Body-length corrected. $l = 975\,\lambda/1000$ (stem-length), $l$. | Body-length as a percentage of body-weight. | Length constant calculated. $k = W^n/l$. $n = 0 \cdot 33$. | Length constant calculated using "best $n$." $n = 0 \cdot 19$. | Body-length calculated. $l = kW^n$. $n = 0 \cdot 19$. $k = 109 \cdot 9$. | Difference between body-length calculated and observed. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | grm. | mm. | grm. | grm. | mm. | | | | mm. | per cent. |
| 1 | 8 | 93,440 | 977 | 1924 | 91,520 | 955 | 1·04 | 21·97 | 109·0 | 963 | 0·83 |
| 2 | 14 | 85,280 | 970 | 1874 | 83,410 | 946 | 1·13 | 22·49 | 109·8 | 946 | 0·00 |
| 3 | 49 | 78,730 | 955 | 1831 | 76,900 | 931 | 1·21 | 22·73 | 109·8 | 932 | 0·11 |
| 4 | 50 | 74,390 | 954 | 1801 | 72,590 | 930 | 1·28 | 23·14 | 110·9 | 922 | 0·87 |
| 5 | 92 | 72,580 | 936 | 1788 | 70,790 | 913 | 1·29 | 22·91 | 109·4 | 918 | 0·54 |
| 6 | 83 | 68,040 | 935 | 1755 | 66,280 | 912 | 1·38 | 23·38 | 110·6 | 906 | 0·66 |
| 7 | 79 | 64,860 | 921 | 1731 | 63,130 | 898 | 1·42 | 23·40 | 110·0 | 898 | 0·00 |
| 8 | 62 | 61,690 | 909 | 1706 | 59,980 | 886 | 1·48 | 23·48 | 109·5 | 889 | 0·34 |
| 9 | 41 | 58,510 | 899 | 1680 | 56,830 | 877 | 1·54 | 23·66 | 109·5 | 880 | 0·34 |
| 10 | 22 | 54,490 | 893 | 1645 | 52,840 | 871 | 1·65 | 24·06 | 110·4 | 868 | 0·35 |
| | 500 | | | | Average ................................ | | | (23·12) | 109·9 | — | 0·40 |
| | | | | | Average, taking into account number of individuals in each group ........... | | | | | | 0·33 |
| Average of groups ..................................... | | | | | 69,430 | 912 | | | | | |
| Average of individuals.................................. | | | | | 67,020 | 907 | | | | | |

is a more complicated matter and concerns the weights. These are not true body-weights, but include the weight of a portion of the clothes, only the coat and boots having been removed in each case before weighing.

In order to arrive if possible at a reliable correction for the weight of this portion of clothing I obtained the weights clothed (partially) and unclothed, as well as other measurements, of 30 undergraduates who happened to be my pupils at the moment. These 30 men are grouped according to weight in Table V. From the average body-weight (part clothed) and the average weights of the part-clothes for the groups a formula for part-clothes was arrived at in the form $c = 69{\cdot}63\ w^{0{\cdot}29}$. This formula fits excellently, the average percentage difference between $c$ calculated by this formula and $c$ observed being only 0·50 per cent., and the mean deviation by the method of least squares 0·61 per cent.

As a further check for the part-clothes their weight was also expressed (see Table V) in percentage of the corresponding body-weight (partly clothed) and the percentage of body-weight (partly clothed) to be assigned to clothes for each group in Table III and Table IV was estimated by interpolation and extrapolation from this series. The values thus obtained were found to agree very satisfactorily with the weights for clothes calculated for the groups in these two Tables by the part-clothes formula. The latter were therefore employed in the Tables in question in order to arrive at the true (unclothed) body-weight for each group.

But the groups in Tables III and IV are not in any case groups which can be expected to exhibit the normal average relation between body-weight and body-length. They cannot be employed as groups in the calculation of a formula for the relationship between these measurements *as growth proceeds*, in the way in which the groups for boys have already been used, for this reason, that they represent a body of individuals measured *at a particular age*. They constitute a selected material, being all University freshmen of an age somewhere between 18 and 20 years, probably for the most part between 18 and 19 years, at the time of observation. Thus the heavier groups do not represent a further stage of growth of the lighter groups. But the whole material simply illustrates the range of measurement from little men to big men at a given age. Hence they differ from a natural growth series by lacking from the groups at the lighter end adolescent boys over 50,000 grm. in weight—say from 15 to 18 years of age—whose inclusion would diminish the average values of $l$ in the lower groups. And they lack at the heavier end all more fully-grown young men of 20 years of age and upwards, whose inclusion would increase the average values of $l$ in the upper groups.

Table V.—Relation between Body-weight (Partly Clothed) and Part Clothes—Data and Calculations.

| No. of group. | No. of individuals in group. | Average body-weight partly clothed, $w$. | Average weight of part clothes, $c$. | Part clothes as a percentage of body-weight partly clothed. | Part clothes constant calculated. $k = w^n/c$ $n = 0 \cdot 29$. | Weight of part clothes calculated. $c = kw^n$. $n = 0 \cdot 29$. $k = 69 \cdot 63$. | Difference between weight of part clothes calculated and observed. | Average body-weight without clothes, W. | Average body-length (stem-length) observed, $l$. |
|---|---|---|---|---|---|---|---|---|---|
| | | grm. | grm. | | | | per cent. | grm. | mm. |
| 1 | 4 | 86,240 | 1883 | 2·18 | 69·74 | 1880 | 0·11 | 84,360 | 922 |
| 2 | 6 | 77,540 | 1817 | 2·34 | 69·39 | 1823 | 0·33 | 75,720 | 934 |
| 3 | 4 | 70,680 | 1793 | 2·54 | 70·36 | 1774 | 1·07 | 68,890 | 915 |
| 4 | 4 | 67,900 | 1748 | 2·57 | 69·37 | 1754 | 0·34 | 66,150 | 901 |
| 5 | 6 | 64,180 | 1711 | 2·67 | 69·04 | 1726 | 0·87 | 62,470 | 872 |
| 6 | 6 | 58,830 | 1688 | 2·87 | 69·85 | 1683 | 0·30 | 57,140 | 894 |
| | 30 | Average .......... | | | 69·63 | — | 0·50 | | |
| | | Average, taking into account number of individuals in each group.......... | | | | | 0·50 | | |
| | | Mean deviation, taking into account number of individuals in each group ... | | | | | 0·61 | | |
| | | Average of individuals .......... | | | | | | 66,540 | 905 |

Accordingly the groups in Tables III and IV do not fit the formula for grouped boys, $l = 23{\cdot}23\ W^{0{\cdot}33}$. They give a series of values for the length constant as calculated from the expression $k = W^{0{\cdot}33}/l$, which increases regularly from a value of about 22 to a value of about 24 as the weight diminishes.

As a matter of fact the calculations summarised in Table III show that the 1000 undergraduates of the first series as grouped give a value for $k$ of 98·22 with a "best $n$" of 0·20 and fit the formula $l = 98{\cdot}22\ W^{0{\cdot}20}$. And the calculations summarised in Table IV for the 500 undergraduates of the second sèries give the value 109·9 for $k$ with a "best $n$" of 0·19.

These facts, however, do not mean that the body-length and body-weight of young men are differently related to each other from the same measurements in boys, and that their bodies are built on a different plan.

The discrepancy depends entirely on the fact that the individuals in question are selected individuals—selected for age. It is well brought out by comparing the figures for body-length as a percentage of body-weight in the heavier boys (Table II) with the corresponding figures for the lighter groups of undergraduates (Tables III and IV).

That it is due entirely to the fact of age-selection follows from the observation that precisely the same phenomenon appears if we select for age among the boys whose measurements have just yielded the formula $l = 23{\cdot}23W^{0{\cdot}33}$. This is seen to be the case in four instances in Table VI where the boys already dealt with have been selected for age, the ages chosen being 11 to 12, 10 to 11, 9 to 10, and 7 to 9 years respectively. In each case the individuals are grouped according to weight, and the value of $k$ is calculated for each group from the average body-weight and average body-length of the group by means of the formula $k = W^{0{\cdot}33}/l$. It is at once evident that in every case by selecting for age and thus excluding from the lower half of each series the heavy boys of ages below the selected age, and from the upper half the lighter boys of ages more advanced, we produce a series of groups which is exactly comparable to the groups exhibited in Tables III and IV.

But though these groups of undergraduates cannot be employed as groups for the calculation of a growth formula, there remains a way in which their measurements can be employed to test the formula for males. This is by averaging the whole material of each series of observations so as to obtain an average individual. These averages are given in Table VII along with the averages of my own 30 undergraduates, and for each series the length constant is calculated from the expression $k = W^{0{\cdot}33}/l$. It happens by a curious chance that the three figures are identical. But the point of importance is

that the value of $k$ thus calculated falls remarkably close to the value of $k$ in the formula for boys, the figures being respectively 23·17 and 23·23.

Table VI.—Boys—Selected for Age—Data and Calculations.

| Age. | Group. | No. in group. | Average body-weight, W. | Average body-length (stem-length), $l$. | Length constant calculated. $k = W^n/l$. $n = 0·33$. |
|---|---|---|---|---|---|
| | | | grm. | mm. | |
| 11 to 12 years ... | 1 | 3 | 40,050 | 754 | 22·83 |
| | 2 | 8 | 36,710 | 738 | 23·00 |
| | 3 | 7 | 34,410 | 735 | 23·40 |
| | 4 | 5 | 33,250 | 733 | 23·60 |
| | 5 | 2 | 32,200 | 732 | 23·82 |
| 10 to 11 years ... | 1 | 4 | 35,360 | 729 | 22·48 |
| | 2 | 5 | 32,870 | 729 | 22·92 |
| | 3 | 5 | 31,530 | 712 | 23·33 |
| | 4 | 4 | 30,140 | 709 | 23·58 |
| | 5 | 4 | 28,130 | 697 | 23·71 |
| 9 to 10 years ... | 1 | 2 | 32,210 | 715 | 23·27 |
| | 2 | 4 | 30,450 | 709 | 23·50 |
| | 3 | 3 | 27,770 | 695 | 23·74 |
| | 4 | 2 | 25,860 | 668 | 23·36 |
| | 5 | 2 | 22,680 | 666 | 24·33 |
| 7 to 9 years ... | 1 | 2 | 31,870 | 703 | 22·96 |
| | 2 | 4 | 27,440 | 676 | 23·18 |
| | 3 | 4 | 25,040 | 670 | 23·69 |
| | 4 | 2 | 23,700 | 668 | 24·05 |

Accordingly it may fairly be concluded that the formula established for boys holds for young adults, and therefore accurately represents the relationship existing between the body-weight and the body-length in males throughout the period of growth from birth to early adult age.

Table VII.—Length Constant calculated for Average Undergraduates.

| Average of individuals. | Average body-weight, W. | Average body-length, $l$. | Length constant calculated. $k = W^n/l$. $n = 0·33$. |
|---|---|---|---|
| | grm. | mm. | |
| First series of 1000 undergraduates ...... | 67,270 | 908 | 23·17 |
| Second series of 500 undergraduates...... | 67,020 | 907 | 23·17 |
| Own series of 30 undergraduates ......... | 66,540 | 905 | 23·17 |

In view of the fact that I have been unable to state with confidence the correction for $l$ in Schuster's undergraduates more precisely than that it

represents a diminution of the observed sitting height by somewhere between 2 and 3 per cent. I have prepared a Table (Table VIII) showing the variations in the calculated values of $k$ when different corrections between these limits are applied to the sitting height, and also when no correction is made.

Table VIII.—Length Constant calculated with Various Corrections for $l$.

| Average under-graduate. | Correction, 2·0 p.c. | | Correction, 2·25 p.c. | | Correction, 2·5 p.c. | | Correction, 2·75 p.c. | | Correction, 3·0 p.c. | | No correction. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Value of | | Value of | | Value of | | Value of | | Value of | | Value of | |
| | $l$. | $k$. | $l$. | $k$. | $l$. | $k$. | $l$. | $k$. | $l$. | $k$. | $l$. | $k$. |
| Series 1 ...... | 912 | 23·27 | 910 | 23·21 | 908 | 23·17 | 905 | 23·09 | 903 | 23·04 | 931 | 23·75 |
| Series 2 ...... | 911 | 23·28 | 909 | 23·23 | 907 | 23·17 | 904 | 23·10 | 902 | 23·04 | 930 | 23·82 |

*Females.*

Table IX (not printed) contains a full record of the data for females, together with the calculated value of the length constant $k$ for each individual deduced from the formula $k = W^n/l$ where $n$ has the value 0·32 as determined in Table X.

In Table X the girls are grouped according to weight in 16 groups. The average body-weight and average body-length for each group is set out, and the figures in the various columns are calculated for each group from the average body-weight and body-length of the group. The body-weights of the groups cover a range in weight from 3834 to 76,430 grm. and show a 20-fold increase from the lightest group to the heaviest.

The "best $n$" for these groups in the formula

$$l = kW^n$$

has the value 0·32, while the value 0·33 is nearly as good. The values of $k$ for the groups are shown in the columns for length constant calculated, and are seen to be free from periodicity. They give an average value for $k$ of 25·60 when $n$ is 0·32, and a value of 23·17 when $n$ is 0·33.

Substituting these values of $n$ and $k$ in the formula $l = kW^n$ the theoretical value of $l$ is calculated for each in the appropriate column. The observed values of $l$ are in good agreement with these calculated values and show an average percentage deviation from them of only 1·41 per cent. when $n$ is 0·32 (1·49 per cent. when $n$ is 0·33).

Table X.—Girls Grouped—Data and Calculations.

| No. of group. | No. of individuals in group. | Average body-weight, W. | Average body-length (stem-length) observed, $l$. | Body-length as a percentage of body-weight. | Length constant calculated. $k = W^n/l$. $n = 0·32$. | Length constant calculated. $k = W^n/l$. $n = 0·33$. | Body-length calculated. $l = kW^n$. $n = 0·32$. $k = 25·60$. | Difference between body-length calculated and observed. $n = 0·32$. $k = 25·60$. | Body-length calculated. $l = kW^n$. $n = 0·33$. $k = 23·17$. | Difference between body-length calculated and observed. $n = 0·33$. $k = 23·17$. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | grm. | mm. | | | | mm. | per cent. | mm. | per cent. |
| 1 | 1 | 76,430 | 940·0 | 1·23 | 25·72 | 22·99 | 935·3 | 0·46 | 947·1 | 0·75 |
| 2 | 7 | 61,030 | 857·9 | 1·41 | 25·24 | 22·61 | 870·2 | 1·41 | 879·2 | 2·42 |
| 3 | 10 | 54,250 | 847·1 | 1·56 | 25·88 | 23·20 | 837·9 | 1·10 | 845·9 | 0·14 |
| 4 | 10 | 53,660 | 831·2 | 1·55 | 25·48 | 22·85 | 835·1 | 0·47 | 842·8 | 1·38 |
| 5 | 9 | 49,390 | 797·2 | 1·61 | 25·11 | 22·54 | 813·2 | 1·92 | 820·0 | 2·73 |
| 6 | 8 | 46,380 | 795·4 | 1·71 | 25·55 | 22·95 | 796·9 | 0·19 | 803·2 | 1·10 |
| 7 | 8 | 41,000 | 770·7 | 1·90 | 25·75 | 23·16 | 766·1 | 0·60 | 771·1 | 0·05 |
| 8 | 4 | 36,990 | 740·5 | 2·00 | 25·57 | 23·01 | 741·3 | 0·11 | 745·4 | 0·66 |
| 9 | 5 | 30,300 | 724·8 | 2·39 | 26·68 | 24·17 | 695·3 | 4·24 | 697·9 | 3·85 |
| 10 | 5 | 24,200 | 659·6 | 2·73 | 26·09 | 23·59 | 647·1 | 1·93 | 647·9 | 1·81 |
| 11 | 4 | 20,170 | 609·3 | 3·02 | 25·55 | 23·13 | 610·5 | 0·20 | 610·2 | 0·15 |
| 12 | 4 | 15,720 | 559·8 | 3·56 | 25·42 | 23·08 | 563·7 | 0·69 | 562·0 | 0·39 |
| 13 | 4 | 10,420 | 485·3 | 4·66 | 25·14 | 22·92 | 494·2 | 1·80 | 490·7 | 1·10 |
| 14 | 3 | 7,876 | 466·0 | 5·92 | 26·34 | 24·09 | 451·9 | 3·12 | 447·4 | 4·16 |
| 15 | 2 | 5,602 | 391·5 | 6·99 | 24·73 | 22·69 | 405·2 | 3·38 | 399·8 | 2·08 |
| 16 | 2 | 3,834 | 355·5 | 9·27 | 25·35 | 23·34 | 358·9 | 0·95 | 352·8 | 1·05 |
| | 86 | Average ................................ | | | 25·60 | 23·17 | — | 1·41 | — | 1·49 |
| | | Average, taking into account number of individuals in each group ........... | | | | | | 1·28 | — | 1·43 |
| | | Mean deviation, taking into account number of individuals in each group ... | | | | | | 1·59 | — | 1·87 |

If account be taken of the number of individuals in each group the average percentage deviation becomes 1·28 when $n$ is 0·32, and 1·43 when $n$ is 0·33. The mean deviation calculated by the method of least squares is, under the same circumstances, 1·59 per cent. when the "best $n$" (0·32) is used, and 1·87 per cent. when $n$ is given the value 0·33.

It follows, therefore, that the stem-length of girls conforms to the formula $l = k\text{W}^n$, where $n$ has the value of 0·32 and the value of $k$ for the grouped girls is 25·60. Moreover, since the grouped girls range in weight from 3834 to 76,000 grm. (12 stone 5 lb.), and in age from two weeks to over 17 years, it may probably be taken that the formula holds for females generally throughout the period of growth from birth to adult age.

For the individual girls the average value of $k$ is 25·58. Its greatest value in the series (28·68) exceeds the average value by 12·12 per cent., and its least value (22·58) falls below the average value by 11·73 per cent.

Using the average value of $k$ (25·58) and the value 0·32 for $n$, the theoretical body-length of each individual has been calculated from the body-weight by means of the formula

$$l = 25{\cdot}58\,\text{W}^{0{\cdot}32}.$$

The figures thus obtained (which from consideration of space have not been tabulated) showed an average percentage difference between the calculated body-length and that actually observed for the individual girls of only 3·02 per cent., and a mean deviation (calculated by the method of least squares) of 4·149 per cent.

*The Difference between the Sexes.*

The formula for the grouped males has been shown to be

$$l = 23{\cdot}23\,\text{W}^{0{\cdot}33}.$$

That for the grouped females is $l = 25{\cdot}60\text{W}^{0{\cdot}32}$. If we desire to work in pounds avoirdupois and inches instead of grammes and millimetres, the formula for males becomes $l = 6{\cdot}91\text{W}^{0{\cdot}33}$, and that for females $l = 7{\cdot}14\,\text{W}^{0{\cdot}32}$.

Now, it has been noted in what has gone before that while the best $n$ for males is 0·33 and the best $n$ for females is 0·32, in each case the values 0·32 and 0·33 are nearly equally good (values on either side of these quantities being distinctly less good).

This fact at once suggested that in taking the power 0·33 for males and 0·32 for females, the sex difference was to some extent exaggerated, the true value of $n$ lying in each case somewhere between 0·32 and 0·33, but nearer 0·32 for females and nearer 0·33 for males.

Accordingly, the "best $n$" was recalculated for each sex to three places of

decimals after regrouping the individuals in a small number of groups to diminish the labour of calculation.

Table XI exhibits the result obtained for males, Group 1 representing the 30 undergraduates measured by myself, and the remaining groups the boys in Table II.

The "best $n$" for males was found to have the value 0·329, with a value for $k$ of 23·45, giving the formula $l = 23{\cdot}45W^{0{\cdot}329}$.

Table XI.—Males re-grouped—Data and Calculations.

| Group. | No. in group. | Average body-weight, W. | Average body-length (stem-length), $l$. | Length constant calculated. $k = W^n/l$. $n = 0{\cdot}329$. | Body-length calculated. $l = kW^n$. $n = 0{\cdot}329$. $k = 23{\cdot}45$. | Difference between body-length calculated and observed. |
|---|---|---|---|---|---|---|
| | | grm. | mm. | | mm. | per cent. |
| 1 | 30 | 66,540 | 905·0 | 23·43 | 905·5 | 0·06 |
| 2 | 23 | 43,100 | 777·1 | 23·21 | 785·0 | 1·01 |
| 3 | 33 | 36,490 | 743·5 | 23·46 | 743·2 | 0·04 |
| 4 | 44 | 32,300 | 723·3 | 23·75 | 713·9 | 1·32 |
| 5 | 29 | 25,280 | 668·4 | 23·80 | 658·6 | 1·49 |
| 6 | 9 | 16,860 | 574·4 | 23·36 | 576·5 | 0·36 |
| 7 | 9 | 9,984 | 479·2 | 23·16 | 485·2 | 1·24 |
| | | Average ...................... | | 23·45 | | 0·79 |
| | | Average, taking into account number of individuals in each group ............................................ | | | | 0·80 |
| | | Mean deviation (for groups)................................... | | | | 1·05 |

Similarly, in Table XII the "best $n$" for females was found to have the value 0·323, with a value for $k$ of 24·80, giving the formula $l = 24{\cdot}80W^{0{\cdot}323}$. It is possible, however, that for the ultimate accurate determination of $n$ to three places of decimals, and of the corresponding values of the length constant, much more extensive series of data than those dealt with here will be required.

But whether $n$ be taken to two places of decimals or to three, it will be seen on inspection that the curve represented by the formula for males and the curve represented by the formula for females have a point of intersection. At this point, and at this point alone, males and females of equal stem-length are of equal weight. The position of the point in question may be ascertained by equating the expressions $23{\cdot}45W^{0{\cdot}329}$ and $24{\cdot}80W^{0{\cdot}323}$, or the expressions $23{\cdot}23W^{0{\cdot}33}$ and $25{\cdot}60W^{0{\cdot}32}$. In the former case it is found to correspond to a body-weight of 11,660 grm. and a stem-length of 510·5 mm.

Table XII.—Girls re-grouped—Data and Calculations.

| Group. | No. in group. | Average body-weight, W. | Average body-length (stem-length), $l$. | Length constant calculated. $k = W^n/l$. $n = 0 \cdot 323$. | Body-length calculated. $l = kW^n$. $n = 0 \cdot 323$. $k = 24 \cdot 80$. | Difference between body-length calculated and observed. |
|---|---|---|---|---|---|---|
| | | grm. | mm. | | mm. | per cent. |
| 1 | 8 | 62,960 | 868·2 | 24·47 | 880·2 | 1·36 |
| 2 | 29 | 52,540 | 826·3 | 24·69 | 830·3 | 0·48 |
| 3 | 20 | 42,350 | 774·5 | 24·80 | 774·5 | 0·00 |
| 4 | 10 | 27,250 | 692·2 | 25·57 | 671·6 | 3·67 |
| 5 | 12 | 15,440 | 551·5 | 24·47 | 559·0 | 1·34 |
| 6 | 7 | 6,070 | 413·1 | 24·78 | 413·5 | 0·10 |
| | | Average | | 24·80 | | 1·16 |
| | | Average, taking into account number of individuals in each group | | | | 0·98 |
| | | Mean deviation (for groups) | | | | 1·86 |

In the latter case it corresponds to a body-weight of 16,220 grm. and a stem-length of 570·4 mm. It is evident that with the data at present in hand it is not practicable to determine the crossing point of the curves more accurately than this. All that can be said at present is that the data for boys and girls show that a crossing takes place somewhere in the region indicated. This may also be seen by comparing the series of body-weights and body-lengths in Table II with those in Table X. Below the point of crossing males are for any given body-length somewhat heavier than females, the difference becoming more marked as the body-length diminishes. Above the crossing the males are for any given body-length somewhat lighter than females, the difference increasing as the body-length increases.

### *Distribution of Errors.*

I have examined the distribution of errors in the foregoing series of observations by noting the percentage deviation of the observed body-length of each individual boy and girl from the theoretical value of the body-length as calculated by the formula appropriate to the sex. From considerations of space the calculations are not tabulated here, but the results of this examination are set out in Table XIII as compared with the theoretical distribution of errors when the mean deviation is calculated by the method of least squares. The number of observations in which the percentage deviation falls within 0·5, 1, 1·5, 2, 3, and 4 times the mean deviation are

expressed in per cent. of the total number of observations. The actual values are in good agreement with the theoretical requirement.

Table XIII.—Distribution of Errors.

| | Theoretical distribution. Percentage of observations. | Distribution for individual boys. Percentage of observations. | Distribution for individual girls. Percentage of observations. |
|---|---|---|---|
| Falling within 0·5 mean deviation ... | 38·3 | 43·5 | 45·3 |
| " 1·0 " ... | 68·3 | 70·7 | 77·9 |
| " 1·5 " ... | 86·6 | 85·7 | 87·2 |
| " 2·0 " ... | 95·4 | 95·9 | 93·0 |
| " 3·0 " ... | 99·7 | 100·0 | 100·0 |
| " 4·0 " ... | 99·99 | 100·0 | 100·0 |

The mean deviation for boys is 3·378.
The mean deviation for girls is 4·149.

The mean deviation for the girls is about 4·2 per cent. That for boys is less, but the girls cover a much wider range in weight than the boys. Taking the worse figure, namely, 4·2, we may at any rate conclude that if the body-length of an individual deviates by as much as 12 per cent. from the theoretical value calculated from the body-weight by means of the appropriate formula the individual is probably abnormal, and that if the deviation reaches 17 per cent. the individual is certainly abnormal.

### *Conclusions.*

1. Throughout the period of growth from birth to adult age the relationship between the body-weight and body-length (stem-length) in Man conforms to the formula $l = k\mathrm{W}^n$.

2. In the male the value of $n$ (to two places of decimals) is 0·33, in the female it is 0·32.

3. The value of the length constant $k$ as determined for grouped individuals is 23·23 for males and 25·60 for females. For the individual boy its average value is 23·33, and for the individual girl 25·58.

4. If the body-length of an individual differ by as much as 17 per cent. from the value calculated by means of the appropriate formula the individual is certainly abnormal; if it differs by 12 per cent. it is probably abnormal.

The relationship between height and weight, and between height and length, is at present under investigation.