Man's Mechanical Efficiency in Work Performance and the Cost of the Movements Involved (Treated Separately).

By J. S. Macdonald, Professor of Physiology, Liverpool.

(Communicated by Prof. C. S. Sherrington, F.R.S. Received May 6, 1914,—received in revised form September 6, 1916.)

Many of the data contained in this paper have been already published* and submitted to a preliminary process of analysis. From the arrangement then made it was seen that the body-weight exercised two separate, and opposing, influences on the heat production associated with muscular work. A certain steady rate of movement was maintained throughout a long series of experiments, and this was complicated to a different degree, in different groups of experiments, with the performance of different, increasing, amounts of mechanical work. When the heat production was comparatively small, in the case of minimal work performance, it was observed to vary directly with the body-weights of the individual subjects. On the other hand, when larger, this variation was less noticeable, and at a certain stage of increase in the performance of work it was found to have disappeared completely. The fact was very definite, so that in four different groups of experiments arranged in order of reference to rising values of mechanical work the total heat productions measured varied in Group A directly with $W^{4/3}$, in Group B with $W^{2/3}$, in Group C with $W^{1/3}$, and in Group D with W^0 (*loc. cit.*, p. 111). No attempt was made at the time, other than contained in a statement of suggestions requiring consideration, to explain this phenomenon, for which course, indeed, an excuse might be found in the labour involved in collecting the information, and the even greater labour of dealing similarly with the very extensive series of measurements underlying the published data. To this problem, then, attention is once more directed in the present paper. In the meantime, these original data have been elaborately and excellently examined by Glazebrook and Dye† in a manner meriting very considerable interest.

Before once more encountering these facts, an explanation of the chief terms utilised may be of advantage, since the mode of experiment and the actual measurements have of necessity to be kept out of sight, and no opportunities arise therefore for an observation of the way in which the measurements are summed to form the total data displayed. Thus, for

* "Studies in the Heat-production Associated with Muscular Work.—Preliminary Communication," 'Roy. Soc. Proc.,' B, vol. 87, p. 96 *et seq.* (1913).

† 'Roy. Soc. Proc.,' B, vol. 87, p. 311 *et seq.* (1914).

example, the main data, throughout termed "heat productions," include frequently a larger quantity of heat than that dissipated from the experimental subject as such, since they include an allowance made for any additional heat stored in his body (an allowance assessed with reference to the rectal temperature), and also include the heat dissipated from the experimental machine (cycle) whenever, and to the same extent as, work is performed upon it by the subject. It is clear that only such sums of the total transformation of energy by the subject are of major physiological interest, as alone equivalent to data obtained from examinations of the exchange of oxygen and carbon dioxide in the concomitant process of respiration, and to data obtained in any other fashion as to the oxidation of material in the body.

Then, again, it is necessary to define the usage of the term "efficiency," since although at the outset of these experiments that usage was as far as possible defined by the very nature of the experiments, occasion has since arisen to utilise the term in an unanticipated way. Thus, originally,* they were arranged to provide a determination of "efficiency" by the comparison of increments in work performance with associated increments of heat production, and that arrangement is very definitely continued in the data under discussion, since the process of experiment was narrowed down to an examination of numbers of individual subjects in two groups of experiments (A and C), differing only from one another by an increment of work and its consequences. Owing to an accident these groups were interrupted, and later, after a complete overhauling of the calorimeter and its apparatus, under improved conditions the same process was renewed, but the results of this later series have been classified under Groups B and D. In either case it was intended to deal finally with the question of efficiency by a comparison of increments observed under similar experimental conditions and it is to be dealt with best by making that comparison where those conditions were at their best, that is, in the later pair of groups (B and D). Utilised in reference to such a comparison, there is not much chance that any misunderstanding can arise as to the meaning of "efficiency." When later the usage of the term is expanded, as first when the efficiency prevalent in the performance of the whole of the external work is considered (as distinguished from the increment) in each of the experiments of Group B and of Group D, even at that stage the term will probably not be misunderstood, since it will be clear that the work done is then being compared to that fraction of the heat production which includes it, and which is associated with its performance and with nothing else, not even

* 'Brit. Assoc. Reports,' p. 289 (1912).

with the accomplishment of the movement in the course of which it is performed. So far the use is simple, and may be expressed in symbols as follows: Let K equal the work done, and $x+K$ the heat production, the incremental efficiency is the ratio $dK/d(x+K)$. When the total efficiency of work performance is dealt with, then x must be considered as split up into fractions $y+z$, where z is equal to ϕK, and where y is "everything else" that has no relation to K, in this case the ratio is $K/(\phi K+K)$. When later "everything else" is dealt with, some simplification ensues, since for the purposes of this paper—for reasons better explained in the conclusion, when the results obtained by such a method of procedure have been examined and found in order—"everything else" which is left of the heat production when ϕK has been subtracted is considered in its entirety as the "cost of movement," no attempt whatever being made to treat it as if again analysable into fractions such as are required by the conventional view that a part of it is "resting metabolism," and only the remainder associated with movement. However, it is in this latter case, in reference to movement, most difficult to estimate the prevalent efficiency, because of the impossibility of directly measuring the work done in movement, but it will be seen that there is promise that an analysis of the "cost of movement"

Data of Groups B and D reprinted in Abbreviated Form from the Preliminary Communication. *

	Date, 1913.	"Stripped weight."	Revs. per minute.	Name.	Heat production in kalories per hour.
	Group B.—Standard rate of movement 60 revs. per minute, associated work 19 kals. per hour.				
I	January 28, 29; February 3, 4, 5	54·6	60	Bennett	193
II	February 17	62·1	59·8	Kemp	218
III	March 3	50·3	60	Gamm	197
IV	March 4	60·5	60	Rae	212
V	March 5	43·7	60	Armstrong	177
	Group D.—Standard rate 60 revs. per min., associated work 56 kals. per hour.				
I	February 18, 26	62·1	60·3	Kemp	350
II	January 27, 30, 31; February 13, 19	54·6	59·8	Bennett	335
III	February 20	60·5	60·4	Rae	347
IV	February 21	60·4	60·5	Hill	345
V	February 24	68·3	60·6	Sharrard	352
VI	February 25, 28	43·7	60·4	Armstrong	346

* 'Roy. Soc. Proc.,' *loc. cit.*, pp. 108, 109.

may somewhat unexpectedly disintegrate that quantity into terms of work done and price paid.

Turning then to the data of the two more recent groups of experiments, B and D (*loc. cit.*, pp. 108, 109), it will be seen that four names, and not more than four, are to be found in both groups, and so provide four individual opportunities for a determination of efficiency by reference to increments of work and heat production. Always speaking of these subjects, Kemp, Rae, Bennett, Armstrong, in the same order, they may be thought of as differing from one another in several ways which seemed to have so little influence on the data that they were not mentioned in the process of preliminary analysis. Thus their heights are different, respectively 168·7, 171·8, 171·2, and 156·4 cm. Their "figures" are different, as may be deduced from a comparison of these heights with the cube roots of their "stripped weights," their respective heights in these terms being, 4·26 $W^{1/3}$, 4·37 $W^{1/3}$, 4·50 $W^{1/3}$, and 4·44 $W^{1/3}$; from which it may be inferred that Bennett was slender (4·5 $W^{1/3}$), whereas Kemp was sturdy (4·26 $W^{1/3}$), and the other two were of intermediate types. The clothes worn weighed respectively, 1·5, 1·0, 3·7, and 3·8 kgrm.; the lighter clothes being the "athletic exercise" garb of two medical students, the heavier the ordinary clothes of two junior laboratory assistants. As a matter of fact, it is difficult to control clothing and the attempt made was limited to the supervision, and provision where necessary, of light shoes. Then the mean rectal temperatures during that part of the experiments (second hour) covered by the data were respectively, 36·6, 37·3, 37·0, and 37·7° C.; the mean surface temperatures, 29·8, 29·9, 31·8, and 34·0° C. More differences, of less importance, could also be mentioned in terms of age, diet, and habit.

First, taking the data from the Tables without correction, that is to say, dealing with all four subjects as if they had each performed exactly the standard amount of work required, equivalent to 56 kalories in D, to 19 kalories in B, and therefore providing an increment of 37 kalories: this increment is then compared with the measured increments of heat production, which differ in the individual cases.

Name.	Weight "stripped."	Heat production.		Increment.	Factors of increment.
		Group D.	Group B.		
	kgrm.				
(1) Kemp	62·1	350	128	132	37 × 3·57
(2) Rae	60·5	347	212	135	37 × 3·65
(3) Bennett	54·6	335	193	142	37 × 3·84
(4) Armstrong	43·7	346	177	169	37 × 4·57

It will be seen that the respective ratios of the number 37 to the numbers 132, 135, 142 and 169 are the individual values of F the efficiency, and realised that the individual values of E (the reciprocal of the "efficiency in work performance") are contained in the last column in the factors 3·57, 3·65, 3·84, and 4·57. The values of these factors expressed in terms of the "stripped weights" of the different subjects are :—

(1) Kemp .... $E = 67\ W^{-0\cdot711}$,
(2) Rae .... $E = 67\ W^{-0\cdot710}$,
(3) Bennett .... $E = 67\ W^{-0\cdot715}$,
(4) Armstrong .... $E = 67\ W^{-0\cdot711}$.

It is of advantage to display these figures in this way prior to making the necessary corrections, since the method of correction is dependent on the accuracy of separate experiments in which measurements were made of the power absorbed by a motor (1) when driving the cycle against the particular brake used in the experiments, and (2), always in the same series of observations, against rope-brakes arranged to entail the same power-absorption. In these experiments it was accurately ascertained that in the neighbourhood of the experimental rate, the power-absorption, and therefore the work done, in the cycling experiments varied with the square of the rate. Aberrations from the standard rate therefore must be regarded as seriously affecting the amount of work done.

Examining the aberrations in rate shown in the tabulated data (Groups B and D, *loc. cit.*), it is found that, with the exception of Bennett, the subjects deviated sensibly and in much the same proportion from this rate in Group D, but maintained it steadily in Group B. The explanation is simple: conducted by the same rhythmical light signal, they were dependent on the clock from which the signal was worked (electrical contact), except Bennett, who preferred to be guided by the tick of a clock inside the calorimeter. These aberrations have been corrected for, in every single datum in the tabulated data, on the precise basis of variation with the square of the rate, and the average correction so obtained entails the following alterations in the statement of work-performance :—

Name.	Actual work, K.		Actual increment, dK.
	K_1, Group B.	K_2, Group D.	
(1) Kemp	18·87	56·56	37·69
(2) Rae	19·00	56·76	37·76
(3) Bennett	19·00	55·59	36·59
(4) Armstrong	19·00	56·67	37·67

As a consequence the values of E formerly given must be altered from (1) 3·57, (2) 3·65, (3) 3·84, (4) 4·57, to (1) 3·50, (2) 3·58, (3) 3·88, (4) 4·49, and the efficiencies prevalent in the four different cases are: (1) 28·6, (2) 28·0, (3) 25·8, and (4) 22·3 per cent. respectively.

Expressed in terms of the "stripped weights" of the different subjects the observed values of E are as follows:—

(1) Kemp	$64{\cdot}57\ W^{-0{\cdot}706}$,
(2) Rae	$64{\cdot}57\ W^{-0{\cdot}703}$,
(3) Bennett	$64{\cdot}57\ W^{-0{\cdot}703}$,
(4) Armstrong	$64{\cdot}57\ W^{-0{\cdot}706}$.

It is therefore clear that efficiency in work-performance is dominated by the value of the body-weight, that it is greater with the greater weight, and therefore that on this account greater weight is an advantage, diminishing heat production. In the particular case just examined it is found, in fact, that

$$F = W^{0{\cdot}705}/64{\cdot}6.$$

However, it is essential, when making this statement, that some indication should be inserted of my opinion that this expression covers only a particular problem and refers, in unmodified form, only to work upon a particular machine. I shall, therefore, venture to insert in the statement a particular function of the body weight, $1/P$ (see (*l*), p. 407), which is of the average value of $W^{0{\cdot}208}/4{\cdot}04$ in these particular subjects, and has a definitely particular average value in this particular case. Modified by its insertion, the efficiency statement is, therefore, as follows:—

$$F = W^{\frac{1}{2}}/16\ P.$$

NOTE.—It is of interest that there is some slight evidence in these data of the influence of factors of secondary importance. Thus in the data of Group D ('Roy. Soc. Proc.,' *loc. cit.*, p. 109) five separate experiments are recorded on Bennett in which his rates of cycling were respectively 59·7, 60·0, 60·1, 59·0, and 60·0. Notwithstanding these differences in rate of movement, and the still larger consequences which ensue in the rate of work performance on the brake (see above), the respective heat-productions are recorded as 338, 332, 336, 333, and 338. Thus, more especially attending to the fourth of these experiments, in which the rate of cycling fell to an annoying degree, Bennett's temperature may explain the fact that the heat production did not similarly fall. At the time he maintained that the cycle-counter was at fault, and that as a fact the rate was properly continued, but his record differs on that day from every other day in the following important points: (1) initial temperature 37·5° C., replacing average 37·1° C.; (2) rectal temperature during experiment 37·33° C. instead of 36·8° C.; (3) surface temperature 33·65° C. instead of 30·81° C., and consequently (*d*) difference of level between rectal and surface temperatures 3·7° C. instead of 6·0° C. In short—and not as an explanation—on that occasion Bennett suffered from, and complained of, a heavy cold. Somewhat similar pathological interest is to be attached to the experiment on Sharrard in the same group.

Up to the present nothing but the increments have been examined, and it is of interest now to observe what order can be obtained on the assumption that this efficiency prevailed similarly in the total performance of external work. Subtracting then the quantity EK_1 from the experiments in Group B, and EK_2 from those in Group D, in both cases utilising the now observed individual values of E and the corrected values of K_1 and K_2, the following residues are obtained.

Residues or Costs of Movement.

Name.	Group B.	Group D.
(1) Kemp	151·9	152·0
(2) Rae	144·1	144·2
(3) Bennett..................	119·2	119·6
(4) Armstrong	91·5	91·7

Although the slightly greater rate of movement (603/600) in Group D might have been expected to produce somewhat larger consequences, to which Bennett should have proved an exception, yet the figures are very satisfactory evidence of the comparability of the two groups of experiments. In view of this evidence these data are accepted as entities separable from the total heat production, and as representing the cost of the underlying movement. Before dealing with them more precisely certain additional data are introduced.

Additional Data (Briscoe).

Doubtless some of the accuracy of the experiments just quoted is to be assigned to the monotonous repetitions of similar experiments, every experiment being a "drill" in handling the very complicated apparatus required, in precisely the most convenient way. The series of experiments now quoted from were of a different type, since large variations in heat production were measured in successive experiments. No doubt they suffer to some slight degree from that fact. Then again they were not all of precisely the same duration, some longer and some shorter than those of the "efficiency groups"; and in addition the calibration of the work done on the cycle was not so satisfactory, nor had it the same direct relation to the actual experimental rates of movement, since in several experiments the subject initiated and maintained his own rate. Once that is said, however, in other points they were similar. The rate of cycling was maintained the same, however much it was varied in different experiments, throughout the whole time of each single experiment. The sets of observations were made at the end of each five-minute period, and none are reckoned in the data that were

taken in the first hour. The "accountancy" of measurements has followed exactly the same rigid plan, and the data now published have been carefully scrutinised again on exactly similar lines. The subject's efficiency, as calculated from his own results, and in agreement with those already considered, is such that the subtraction necessary to remove the moiety of heat production associated with external work performance is 3·85 K. Removing these fractions the "cost of movement" is left, and in this case it represents that cost at definitely different rates of movement.

E. J. Briscoe, 1912. "Stripped weight" 55·8 kgrm. E = 3·85.

Date.	V. Revs. per minute.	K. Work.	H. Heat production.	EK. Subtraction.	Q = H − EK. Residue.
1912.					
(1) May 24	40	21	159	81	79
*(2) May 14	60	13	167	50	117
*(3) May 17	60	26	212	100	112
*(4) May 20	60	34·5	244	133	111
*(5) May 22	60	42·5	286	164	122
(6) May 7	72	49	341	189	152
April 25	74	16	222	62	160
May 21	80	73	456	281	175
†March 28	97 (?)	10	291	40	251
†March 29	98 (?)	11	316	42	274

* Data in British Association Report, 1912, p. 289, numbered there as 2, 3, 4, 5.

† The query by the side of the revolution rate will be understood to refer to the difficulty of maintaining these fast rates in a perfectly uniform way, there always being a tendency for groups of faster to succeed groups of slower movements.

These Briscoe data have been given in the same form as all the data up to the present, and may be compared at once with them, but for the purposes of the later part of this paper they are now changed in form. Kalories per hour are now changed into small calories per second, revolutions per minute into "per second," and doubling this revolution rate, so taking account of the two complete leg-movements associated with each revolution of the cycle pedals, they are presented as "strides per second."

Another change may also be noticed from this point onwards, namely, the large number of apparently significant figures in which the rates of movement are expressed (thus, 2·667 per sec.), but this method has proved of value, in so far as these figures are submitted to speculative arrangement always in the same definite form in which they are given.

Table I.—Briscoe.

V = strides per second. Q = residue or "cost of movement."

V	1·333	2·000	2·400	2·466	2·667	3·200	3·267
log V	0·124	0·301	0·380	0·392	0·426	0·505	0·514
Q	22	32	42	44	49	70	76
log Q	1·342	1·505	1·623	1·643	1·690	1·845	1·881

Further Additional Data (Douglas).

To complete the range of data essential to a full consideration of the "cost of movement," I have, of necessity, had to consult the data given by other investigators, and have chosen as most suitable for my purpose equivalent data published by Douglas and Haldane,* which are in their original form measurements of the oxygen absorbed and the carbon dioxide produced by Douglas when walking on the grass at Oxford at rates varied from 2 to 5 miles per hour. These data have been converted into calories per second, following the Zuntz and Schumburg method in exact detail. The method may be readily ascertained from Benedict and Cathcart's description.† This done, I have converted "miles per hour" into "strides per second" in an arbitrary fashion, since the stride-length of Douglas is not given. For this purpose I have credited him with a length of stride of 0·837 metre (33 inches), thus allowing 0·533 stride per sec. as the equivalent of 1 mile per hour.

In dealing with these data no subtraction whatever has been made. The view is taken that the process of "walking on the grass at Oxford" is not associated with any other than a negligible amount of external work performance. The whole heat production is thus in this case treated as "cost of movement," and is dealt with as if caused by phenomena completely resembling those underlying the residue of heat production in Briscoe's case, completely, that is to say in everything but magnitude.

Table II.—Douglas Walking on the Grass at Oxford.

V = strides per second. H = heat production in calories per second.

V	2·667	2·400	2·133	1·600	1·067
log V	0·426	0·380	0·329	0·204	0·028
H...........................	210	164	131	88·5	63
log H	2·322	2·215	2·118	1·947	1·799

* 'Journ. Physiol.,' vol. 45, p. 235 *et seq.* (1912); also quoted in 'Phil. Trans.,' B, vol. 203.

† 'Muscular Work—A Metabolic Study, with Special Reference to the Efficiency of the Human Body as a Machine,' Carnegie Institution, Washington, 1913, p. 33.

The Cost of Movement.

Formulation of the Cost.

It is a matter of common knowledge that the "cost of movement" increases with increasing body weight. It is therefore different from the cost of work performance, for which heavier individuals are chosen for the continued performance of heavy work. That the fact is a very definite one may be seen at once from the residues of my four cycling subjects on p. 401. For brevity those residues (Group B) are at once expressed in terms of the individual "stripped weights" of the four subjects subsequent to conversion from kalories per hour into small calories per second (division by 3·6).

(1) Kemp .... $Q = 0{\cdot}1114\ W^{1{\cdot}438}$,
(2) Rae .... $Q = 0{\cdot}1096\ W^{1{\cdot}438}$,
(3) Bennett .... $Q = 0{\cdot}1053\ W^{1{\cdot}438}$,
(4) Armstrong .... $Q = 0{\cdot}1114\ W^{1{\cdot}438}$,

and at the same rate of movement (see Table I, 2 strides per second),

(5) Briscoe .... $Q = 0{\cdot}0991\ W^{1{\cdot}438}$.

Taking the average of the first four, obtained under the same experimental conditions, it may be said that at this rate of movement (one cycle revolution, or two strides per second),

$$Q = 0{\cdot}1094\ W^{1{\cdot}438}. \qquad (a)$$

And now turning to the influence of the rate of movement on this cost, it is also certainly very definite, even if complex. Thus the whole of the Douglas data (Table II) may reasonably be considered as expressible in the following formula:—

$$H = 52{\cdot}37\ (1{\cdot}475\ V)^{0{\cdot}380\,V}, \qquad (b)$$

as is shown by a comparison of the data,

(1) 210, 164, 131, 88·5, 63,

with the values deduced from the formula,

(2) 210, 166, 133, 88, 63.

As a matter of fact, it is possible to make a choice between this formula and others of a somewhat similar type, equally, if not more, satisfactory for this purpose, but this formula has been deliberately chosen as of a certain greater rigidity of type which is of value when comparisions are made with attempts to formulate the cost of other movements. Thus, for example, the whole of the Briscoe data can be reasonably held to be expressible in an exactly similar, and similarly rigid, formula as follows:—

$$Q = 16{\cdot}45\ (1{\cdot}783\ V)^{0{\cdot}260\,V}, \qquad (c)$$

as is shown by a comparison of the data

(1) 22, 32, 42, 44, 49, 70, 76

with the values deduced from the formula

(2) 22, 32, 41, 42, 48, 70, 73.

The agreement is certainly least where the experimental data are necessarily fallible.

In reference to the influence of "rate" upon the cost of movement, and in exact explanation of the nature of the formulæ, it may be said at once that the point of greatest importance lies in the fact that the cost per stride is least at a certain intermediate rate of movement, and therefore also the cost of progression is least at the same intermediate rate. In both of these formulæ that fact is placed in unusual prominence by the direct insertion of this economical rate in a definite position of importance in the formulæ. The cost per stride in Douglas' case, that is to say, the value of HV^{-1}, is least when V has the value 1·475, and in Briscoe's case the value 1·783. Speaking of the particular value of V as in each case P, then the two formulæ have the resemblance shown below:—

$$(b) \text{ Douglas } \ldots\ldots\ldots\ldots\ldots \quad H = 52{\cdot}37\,(PV)^{0{\cdot}380\,V},$$
$$(c) \text{ Briscoe } \ldots\ldots\ldots\ldots\ldots \quad Q = 16{\cdot}45\,(PV)^{0{\cdot}260\,V}.$$

Nor is this the end of the resemblance, as may be seen from the considerations stated below.

Digressing a moment, but as briefly as possible, it may be stated as an axiom that, with regard to every formula of the general type, $R = x\,(yV)^{zV}$, the value of V at which RV^{-1} is minimal is determined by the relation

$$z\,V_1\,(\log_e V_1 + \log_e y + 1) = 1.$$

In this particular case $P = V_1 = y$, and therefore

$$z\,P\,(\log_e P + \log_e P + 1) = 1,$$

therefore $$z = 1/[P\,(2\log_e P + 1)]. \qquad (d)$$

That is to say that, in this particular rigid formula, z is also a function of P, and may be represented by P′; and this is exactly true in the two formulæ given: in the one case 0·380 has this relation to 1·475, and in the other 0·260 to 1·783. The present resemblance between the two formulæ is therefore seen in the fact they may be both written as follows:—

$$(b) \text{ Douglas } \ldots\ldots\ldots\ldots\ldots \quad H = 52{\cdot}37\,(PV)^{P'V},$$
$$(c) \text{ Briscoe } \ldots\ldots\ldots\ldots\ldots \quad Q = 16{\cdot}45\,(PV)^{P'V}.$$

If it could be shown, then, that 52·37 in the one case, or, as it may be termed, T in formula (b), is also a function of 1·475, and the same

function as is 16·45, the T in formula (*c*), of 1·783, then H would be a function of P, Q also a function of P, and the functions would be identical in character. This actually seems to be the case, as is shown incidentally in the following method of dealing with the two formulæ, which is of use later in assessing values of P in additional individual cases (see (*l*), p. 407).

(*b*) Douglas $H = T(PV)^{P'V} = 52{\cdot}37\,(1{\cdot}475\,V)^{0{\cdot}380\,V}$,

therefore $\log H = 1{\cdot}719 + 0{\cdot}380\,V \log V + 0{\cdot}064\,V$,

$= 1{\cdot}719\,(1 + 0{\cdot}221\,V \log V) + V \log 1{\cdot}16$,

therefore $H = 1{\cdot}16^V T^{1+0{\cdot}221\,V \log V}$.

It will be seen that this new form of statement is permissible, inasmuch as $P^{P'}$ is approximately equal to 1·16, and because

$$P' = 0{\cdot}221 \times 1{\cdot}719 = 0{\cdot}221 \log T. \qquad (e)$$

(*c*) Briscoe $Q = T(PV)^{P'V} = 16{\cdot}45\,(1{\cdot}783)^{0{\cdot}260\,V}$,

therefore $\log Q = 1{\cdot}216 + 0{\cdot}260\,V \log V + 0{\cdot}065\,V$,

$= 1{\cdot}216\,(1 + 0{\cdot}213\,V \log V) + V \log 1{\cdot}16.$

$= (1 + 0{\cdot}213\,V \log V) \log T + V \log 1{\cdot}16.$

therefore $Q = 1{\cdot}16^V T^{1+0{\cdot}213\,V \log V}$.

This statement, similar to that given in the case of Douglas, is again also permissible because $P^{P'}$ is approximately equal to 1·16, and because

$$P' = 0{\cdot}213 \times 1{\cdot}216 = 0{\cdot}213 \log T. \qquad (f)$$

Now when (*e*) is compared with (*f*), with an appreciation of the fact that the quantities of heat dealt with in the two cases are in the ratio of 210/49 at the same common value for V, (V = 2·667), also of the ease with which it would be possible to adjust the small difference thus revealed without modifying the formulæ probably in any other way than to make them still more applicable to the data than they are at present, it will be granted that (*e*) and (*f*) reveal T as the same identical function of P. T may then be written as P″. It follows that the formulæ are completely identical in form, and may either be written as follows :—

$$H \text{ or } Q = P''(PV)^{P'V} = \phi(P, V).$$

The only difference, then, between the cost of walking, on the one hand, and the cost of cycling on the other, is to be sought in the different magnitude of P in the two cases, and in either case the cost of movement at every other rate, including that at the "economical rate," where this cost is least, may apparently be anticipated after an examination of the cost at any one definitely maintained rate.

It is now useful to define T in terms of P, and a possible procedure is as follows :—

Let $T = 1{\cdot}17\mu$, therefore (Douglas) $\mu_1 = 44{\cdot}76$, (Briscoe) $\mu_2 = 14{\cdot}06$.

Let $\mu = y P^z$ and therefore $\log \mu = \log y + z \log P$,

therefore (Douglas) $\quad 1{\cdot}651 = \log y + 0{\cdot}169 z, \qquad (1)$

(Briscoe) $\quad 1{\cdot}148 = \log y + 0{\cdot}251 z. \qquad (2)$

From these equations

$$z = -6{\cdot}134, \qquad \text{and } \log y = 2{\cdot}687,$$

therefore also $\quad y = 486{\cdot}5 = e^{6{\cdot}187}$,

therefore $\quad \mu = e^{6{\cdot}187}/P^{6{\cdot}134}$,

and $\quad T = 1{\cdot}17\mu = 1{\cdot}17\, e^{6{\cdot}187}/P^{6{\cdot}134}. \qquad (g)$

Introducing this expression for T in the general equation

$$H^* = 1{\cdot}17\,(e^{6{\cdot}187}/P^{6{\cdot}134})\,(PV)^{P'V},$$

therefore $\quad 0{\cdot}426\ H = \tfrac{1}{2}\,(e^{6{\cdot}187}/P^{6{\cdot}134})\,(PV)^{P'V}. \qquad (h)$

The suggestion implied in the form (*h*) is obvious, and may be briefly expressed by saying that the expression

$$0{\cdot}426\,H = E'\,(\tfrac{1}{2} m u^2) = \tfrac{1}{2}\,(mE')\,u^2, \qquad (j)$$

would represent a rational formula, in which E′ was the reciprocal of the efficiency in movement, also that there is some promise shown in (*h*) of a final statement in this rational form.

Under the impression that this promise is sufficient to permit immediate examination of the formula from such a point of view, I shall venture to speak of part of the formula as the possible "(mE')" factor.

The "(mE′)" Factor.

An attempt to ascertain the relation between $e^{6{\cdot}187}/P^{6{\cdot}134}$ and the mass, and at the same time to express $1/P$ in terms of W, may be made very simply by utilising the relation found in the case of the four cycling subjects, $Q = 0{\cdot}1094\ W^{1{\cdot}438}$. It will be remembered that for both Douglas and Briscoe, and therefore inferentially the cycling subjects, $Q = 1{\cdot}16^V T^{1+{\cdot}213 V \log V}$. It is true there was a slight difference in Douglas' case, but it was even then of minimal value, and there can be little hesitation in applying the exact form of Briscoe's statement to the subjects examined under similar conditions. In their case, since $V = 2$, the latter expression becomes $Q = 1{\cdot}346\ T^{1{\cdot}128}$,

$$0{\cdot}1094\,W^{1{\cdot}438} = Q = 1{\cdot}346\,T^{1{\cdot}128},$$

* Or Q.

therefore $$T = W^{1 \cdot 275}/9 \cdot 284 = 1 \cdot 17\mu,$$

therefore $$\mu = W^{1 \cdot 275}/10 \cdot 86. \tag{k}$$

But

$$\mu = e^{6 \cdot 187}/P^{6 \cdot 134} = W^{1 \cdot 275}/10 \cdot 86 \quad \text{therefore} \quad P_c{}^{*} = 4 \cdot 042/W^{0 \cdot 208}. \tag{l}$$

Therefore the mE' factor and P have both been expressed in terms of the "stripped weight." However, since the mass in motion includes the clothes, it seems essential at this point to introduce the "clothed weight"; and an interesting way of doing so is to determine the values of P for Kemp and Armstrong, from the general expression (l) given above; and then convert the values so found into terms of the clothed weight. Substituting 62·1 and 43·7 for W in (l), it follows that in Kemp's case $P = 1 \cdot 714$, and in Armstrong's case $P = 1 \cdot 846$. Now their clothed weights (for weight of clothes, see p. 397) were 63·1 and 47·5 kgrm. respectively; terming this weight W_2, in both cases the given values of P_c may be expressed as

$$P_c = 4 \cdot 88/W_2{}^{0 \cdot 252}; \tag{m}$$

substituting this new value for P_c in the "(mE')" factor,

$$e^{6 \cdot 187}/P^{6 \cdot 134} = (W_2/9 \cdot 82)^{1 \cdot 546} = (W_2/g)^{1 \cdot 546},$$
$$= m^{1 \cdot 546},$$

therefore $$mE' = m^{0 \cdot 546} \times m. \tag{n}$$

Recognising now with some certainty the fact that this is indeed a "mass factor," even if qualified by something in the nature of E′, it is safe to conclude that either $1/P$ or $1/P^2$ is a unit of length. The conclusion may be said to follow at once from the general cubical nature of the factor, which may, indeed, be described as $(e^2/P^2)^3$ qualified by a correction for clothing (difference between $e^{6 \cdot 187}$ and $e^{6 \cdot 134}$) and a correction for density (difference between 6·134 and 6·000). Having arrived at this conclusion, however, it is reasonable to be prepared for several different relationships to the whole mass of the body resulting from temporary alterations in this individual unit of length, not only in reference to the individual mass, but also to the degree of shortening or extension of the body mass associated with individual movements. In short, it is reasonable to infer that the comparison between the value of P found for Briscoe and that found for Douglas is not assignable merely to different body weights, but also to an essential distinction between P_c and P_w, the "cycling" and "walking" values of P respectively. Taking this view, I have considered it not unwise to assume that whereas P_c is related

* P_c denotes the "cycling value" of P.

to $W_2^{1/4}$ as shown, it is extremely likely that in the fully extended dimension of the body utilised in walking, P_w will be found rather related to that function of the weight which is of so great comparative value in connection with the linear dimensions of the body, namely, the cube root of the weight. Considering the mass factor, $e^{6 \cdot 187} P^{-6 \cdot 134}$, with this possibility in mind, it would seem as if under such circumstances the whole factor would become m^2. This view may be tested immediately, since it should be possible to calculate Douglas' weight on the assumption that in his case $\mu = (W_2/g)^2$.

Thus $\qquad \mu = e^{6 \cdot 187} P^{-6 \cdot 134} = T/1 \cdot 17 = 44 \cdot 76$ (Douglas),

therefore $\qquad 44 \cdot 76 = (W_2/g)^2$ and therefore $W_2 = 65 \cdot 6$ kgrm.

But since his weight is given in 1910* as 65 kgrm., the assumption would seem to be reasonably justified. It is probable then that, whereas $P_c = 4 \cdot 88/W_2^{0 \cdot 252}$, $P_w = 5 \cdot 77/W_2^{0 \cdot 326}$, so that in Douglas' case the observed value of 1·5 in the case of walking movement would probably correspond with a value of 1·7 in reference to cycling movement.

The Velocity Factor $(PV)^{P'V}$.

In the case of "Douglas walking," it is possible to compare with some interest the square root of the velocity factor on the one hand with the actual horizontal velocity or rate of progression on the other, and perhaps this is best done at first in reference to the most important "economical rate" P, when the cost of stride is least. At the rate P per second, since $V = P$, therefore $(PV)^{P'V} = P^{2PP'} = (1 \cdot 475)^{1 \cdot 121} = 1 \cdot 546 = (1 \cdot 243)^2$. Again, since the length of a stride is 0·837 metre, the horizontal velocity at the rate P is 0·837 P, and is therefore 1·235 metres per second. For brevity, using the term f in place of $(PV)^{\frac{1}{2}P'V}$, it is seen that there is no great difference at this rate between v and f.

In general, the relation between v and f is such that the line v intersects the curve f at two points, where $V = 1 \cdot 5$ and where $V = 1 \cdot 5^3$; that is to say that, at rates not very different from P and P^3 respectively, v is equal to f. At intermediate values of V the horizontal velocity is slightly the greater quantity, the maximal difference of 0·213 metre per second being found at the rate $1 \cdot 5^2$, that is to say, at a rate not very different from P^2. Beyond these points on either side of P and P^3 the curve rapidly falls away from the line, so that f becomes much greater than v.

In short, although v is not a tangent to f, yet $v + 0 \cdot 213$ is such a tangent at the point where $V = 1 \cdot 5^2$. There does not seem, under these

* 'Journ. Physiol.,' vol. 40, p. 235.

circumstances, any reason to hesitate before suggesting the possibility that f, or $(PV)^{\frac{1}{2}P/V}$, may finally be shown to have some definite relation to the acceleration or sum of accelerations responsible for the quasi-pendular movements from which the horizontal velocity is derived.

As a summary to the sections dealing with mE', and with the velocity factor, it may be stated, then, as not improbable that the empirical formula $0{\cdot}426H = \frac{1}{2}\phi(1/P)(PV)^{P/V}$, may finally be arranged in rational form in some such manner as

$$0{\cdot}426\,H = \tfrac{1}{2}\,mE'f^2.$$

Conclusion.

Attention is drawn to the fact that there appears to be some definite order in the heat productions dealt with as "cost of movement" when no allowance is made for synchronous "cost of rest." The order developed by dealing with the facts in this way is significant, in my opinion, of the influence of that control exerted by the central nervous system in arranging the phenomena of a moment to correspond with the requirements of that moment. In my view "rest" is an entity, and not devoid of a dynamic fraction, "movement" is again an entity of a different type, in which the dynamic fraction is in the forefront, and on this view everything occurring during movement is related to movement. In short, the view is held that there is no "rest" in movement, and apparently with some justification. It is also held, and apparently has indeed been shown, that the cost of movement is identically the same whether the work performed by its means is large or small.

Attention is also drawn to the importance of an "economical rate" in movement as the phenomenon of major interest, and as decided by relationships to bodily dimensions of an exact kind. There is no sign, indeed, that any other circumstances need be considered than mass and length in this connection.

Then, as to the "efficiency" prevalent in the performance of movement, it would seem to *vary inversely* with the mass in motion, but the fact that this is the case suggests at once the conception that this efficiency (so far as it can be examined) is not a genuine efficiency, but is due to the complication of a constant, perhaps, indeed, an absolute efficiency (100 per cent.), by unknown internal resistance directly proportional to the mass engaged in accomplishing visible external work again proportional to the mass, so that the cost varies with the square of the mass.

A similar point is even still more evident with regard to the "efficiency of work performance." In this case a different mechanical problem, leverage

perhaps as contrasted with quasi-pendular movement, exhibits an efficiency *varying directly* with the weight or mass in such a way that the cost is diminished by increase of mass, as if the different mechanical considerations involved in this separable process arranged the mass in the other pan of the scales of cost.

The Typical Form of the Cochlea and its Variations.

By Henry J. Watt.

(Communicated by Prof. D. Noël Paton, F.R.S. Received October 3, 1916.)

The work of this paper is based upon the photographic and descriptive material presented by Dr. A. A. Gray in his two volumes on 'The Labyrinth of Animals,' published by J. and A. Churchill, London, in 1907 and 1908.* I have succeeded in extracting from that impressive mass of material definite results that seem to be of some importance.

The dimensions of the cochlea measured by Dr. Gray are: (1) the diameter of the lowest whorl and (2) of the second whorl ("taken in a plane which passes vertically through the apex of the cochlea and the anterior margin of the round window"); (3) the diameter of the tube of the cochlea in front of the round window; (4) the major axis of the oval window: (5) the slant height of the cochlea ("the distance from the upper margin of the round window to the apex of the organ"); and (6) the number of turns of the cochlea.

I found it desirable to add to these a measurement of the total length of the basilar membrane. That must surely represent more closely and directly than anything else the pitch-range of hearing. Fortunately, a close study of Gray's wonderful photographs showed that an approximate measurement of the length of the basilar membrane (as of the outside edge of the cochlear tube) could be got from them. The symmetrical shape of the cochlea makes it possible to measure the diameters of the successive whorls, no matter from what angle the photograph was taken. (The reader must consult Gray's pictures.) With the help of Gray's measurements of the diameters of the first and second whorls, by close attention to the consistency of these with the dimensions visible in the photograph, and by a careful comparison of the different photographs showing the cochleas of

* *Cf.* also 'Roy. Soc. Proc.,' B, vol. 78, p. 284 ff. (1906), and B, vol. 80, p. 507 ff. (1908).